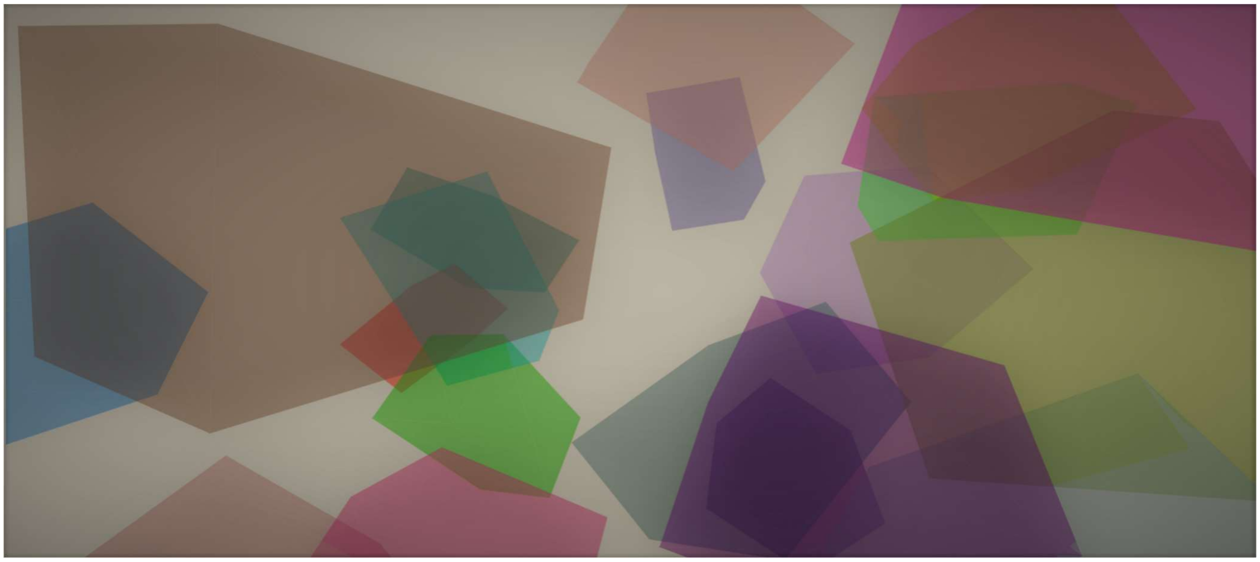

ANALYSIS OF SCORING AND RATING MODELS USING NEURAL NETWORKS

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Abstract: *This reasearch paper investigates an approach for analysis of an established system to determine credit rating and scoring, according to regulatory requirements. For this purpose, a model of a neural network is used, on which the realised logic is transferred. According to the properties of the model, sensitivities, significance, independency and other parameters of the input factors are determined.*

Key words: *credit rating, scoring, regulatory requirements, analysis of the factors.*

Credit scoring represents an assessment of the borrower's credit worthiness in future time periods. It is expressed numerically, and afterwards rating classes are defined from it, using specific rating scales which are used for internal or regulatory models for credit risk assessment.

For credit scoring determination statistical methods are used to obtain information and dependencies from the borrower's primary data known at the time of the scoring calculation. The systems for scoring and rating are used to forecast the conditions in which the borrowers will be able to settle their debts or their default.

The data used for such assessments could be for example: age, residence, net income, financial assets, credit size, credit duration, etc. This data can be analyzed using mathematical and statistical models. The more complete the data history is, the more accurate the models will be. Scoring and rating systems are basically classifiers that determine the scoring or rating class of a borrower based on their primary data. A variety of approaches are used for this purpose such as hierarhical systems of rules, algebraic expressions, decision tables, graphs, etc.

This research paper is based on the assumption that the logic of the scoring systems is established. The aim is to propose methods for analysis, determination of characteristics and validation of the logic, data and the credit scoring, using methods based on neural networks. The analysis is performed according to the following principles:

- The logic of the scoring model is transferred to the neural network:
  Borrowers with their input data and already known scoring results are treated as examples of scorings and used for the neural network with a predetermined structure to be trained.

- The neural network is analyzed instead of the original scoring system:
  After the training the neural network has gained certain standard knowledge in terms of internal objects and weighted connections between them. Thus, it represents a standardized alternative model which can be analyzed.

- Analysis and assessment of charachteristics of the scoring model:
  Analysis of charachteristics of the scoring model are required by regulatory authorities, according to Regulation (EU) № 575/2013 and refer to the validation of the scoring method, determining the factors' signifficance, independence, sensitivity and influence, as well as the orthogonalization of the scoring structure, amongst other things. Based on these characteristics, one can ascertain the reliability of the model. These characteristics make the scoring model's reliability the most important element of credit risk assessment.

1. Basic Concepts of Neural Networks

Neural networks are defined as statistical models for information processing. Different types of neural networks exist and those used for the scoring systems are supervised neural networks which work in two phases. In the first phase, internal modifications and calibration are carried out according to the input and output data (representing the training set for the neural network). Each example consists of input data and corresponding results. All examples pass through the neural network many times, with changes in the soft-computing structure being carried out iteratively: some connections are strengthened, others weakened. Here, input data represent the borrower's data and results represent the scoring value. In the second phase, input data are processed and still unknown results are generated. This is done by adding the input data to the already trained neural network, for which there are no results available. The results are generated by the neural network using already defined internal structure.

Artificial neural networks are models of the biological neural network and thus have certain similar features. The training of the biological neural networks is achieved by multiple chemical changes in the synaptic connections between neurons with spatial and temporal activity. During the training, information is accumulated in the synapses in the form of concentrated chemical substances. Some of the basic characteristics of these models are as follows:

- Training and Adaption

  The "training" phase must take place prior to the "actual work" phase. The training is a time-consuming and complicated process, in which a specific algorithm is followed in order for the appropriate values of the connections strenght between the individual elements to be found. These connections are changed by each example based on the information incorporated into it. It is possible to add new examples after completing the training, so that one can continue with the next training level. This, however, may cause the old samples to be "forgotten", if they are not presented anymore. Such an effect may be useful and can be used to realize the models evolution on the time axis. The training examples can be considered as a statistical sample of an infinite or a very large data set. Thus the more representative this sample is, the more adequate the training will be. A training can be regarded as an optimization process in which the system is calibrated according to the sample.

- Generalization

  Generalization is a feature that describes the possibility of making adequate decisions in unknown situations based on previous experience. The process of generating new information is based on the principle that similar incentives produce similar responses. If the experience of the neural network is greater, than it is more probable to generate adequate responses in unknown situations. This, however, may also make the neural network structure more complicated and the model, like a black box, more difficult to understand. Moreover, the neural network may over-adapt only to the presented training examples (which is called overfitting), but its results in the generation of new data could be significantly poorer. For this reason, sometimes a part of the training set is separated into a validating set, which is then used to test the network's ability to generate an adequate response for new, unfamiliar data.

- Spatial information processing

  The principle according to which neural networks function, both in the training phase and in the phase of generating new information, is based on mathematical methods and computation of vector-matrix equations. During this process, some specific subsets of elements or individual elements are specialized as reacting to certain incentives and can be considered as representatives for specific concepts, depending on the task presented. They represent certain templates, features, functions, tasks, or goals. After the training, the connections of the neural network structure are not changed, and they can be further analyzed so that some of them that are with insignificant influence can be removed.

- Reliability

  Removing certain processing elements (neurons) often has no major influence on the working of the model. This allows some of the connections and elements with insignificant influence to be excluded, which simplifies the model and facilitates its analysis. This is important in case of searching for incorporated concepts and conclusions about the importance of the factors in the examples available.

The presented analysis here is based on the main features described above.

2. Experimental Scenario for Modelling

The existing data used for credit scoring analysis contains several factors that provide information about the borrowers, such as: name, age, salary, savings, other income, net income, properties price in euro, properties area in squared meters, number of persons in the household, education level, occupation, etc. Some factors do not participate in the scoring calculation because of one or several of the following reasons:

- They do not provide the required useful information for the functioning of the model
  Factor "borrower's name", e.g., does not affect the work of the model.

- They cannot be represented as numeric values
  If the data is represented as character strings or other, different from numerical, data types, they must be converted into numerical values in some way (see also section Calibration below). The model uses only numerical data and if some factors cannot be represented numerically, they cannot be taken into consideration.

- Lack of sufficient data
  If the existing non-zero data, e.g., in the database for the factor "other incomes of the borrower" is about 5%, then it does not provide useful information for the model.

- Certain factors contain other factors
  The "salary" factor, e.g., is included in the "net income" factor.

Calibration of input factors

The influence of input factors must be calibrated using linear and non-linear (saturation inclusive) *S* and *Z* functions (Fig. 1), prior to being presented to the neural network.

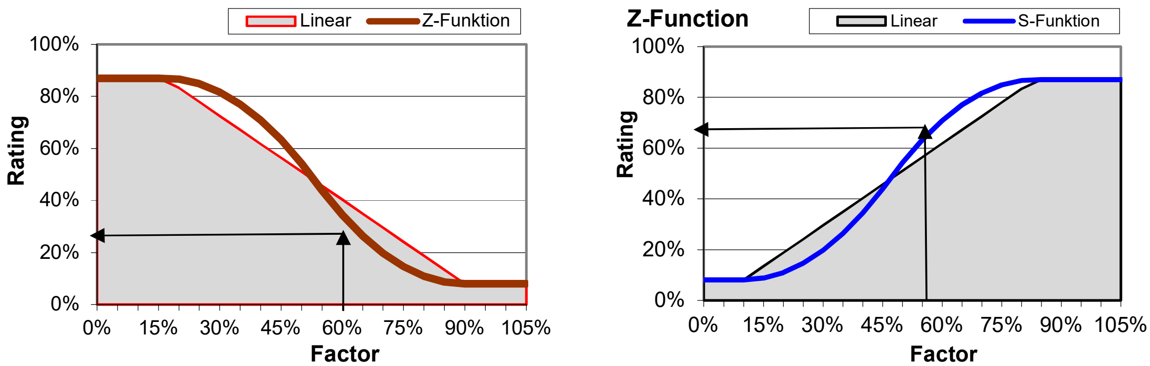


Fig. 1. Calibration of input factors (S- and Z-functions)

The *S*- and *Z*- functions transform the values of the factors into calibrated values, from 0% to 100%, as follows:

S-Function: Y = S (X, a, b); a, b – constant, depending on the factor's range

for $X \le a$: $S(X, a, b) = 0$;

for $X > b$: $S(X, a, b) = 1$

for $a<X \le (a + b)/2$: $S(X, a, b) = 2(X-a)/(b-a)$;

for $(a + b)/2<X \le b$: $S(X, a, b) = 1-2(b-X)/(b-a)$

Z-Function: Y = Z (X, a, b)

$Z (X, a, b) = 1 – S (X, a, b)$

The structure of the neural network

In the results presented here, after the phase of factors selection and processing, 22 factors are selected that are used for the construction of the neural network. Thereafter, it is trained with several hundred records of borrowers' data. The network structure chosen here (Fig. 2) is of a multi-layer perceptron, where the number of input neurons corresponds to the number of factors. There is only one output neuron corresponding to the borrower's scoring value. The neural network structure also contains one hidden layer, the size of which is determined depending on the number of the training examples. Connections between layers (the matrices *Wji* and *Wkj*) are represented as matrices that contain the connections' weights.

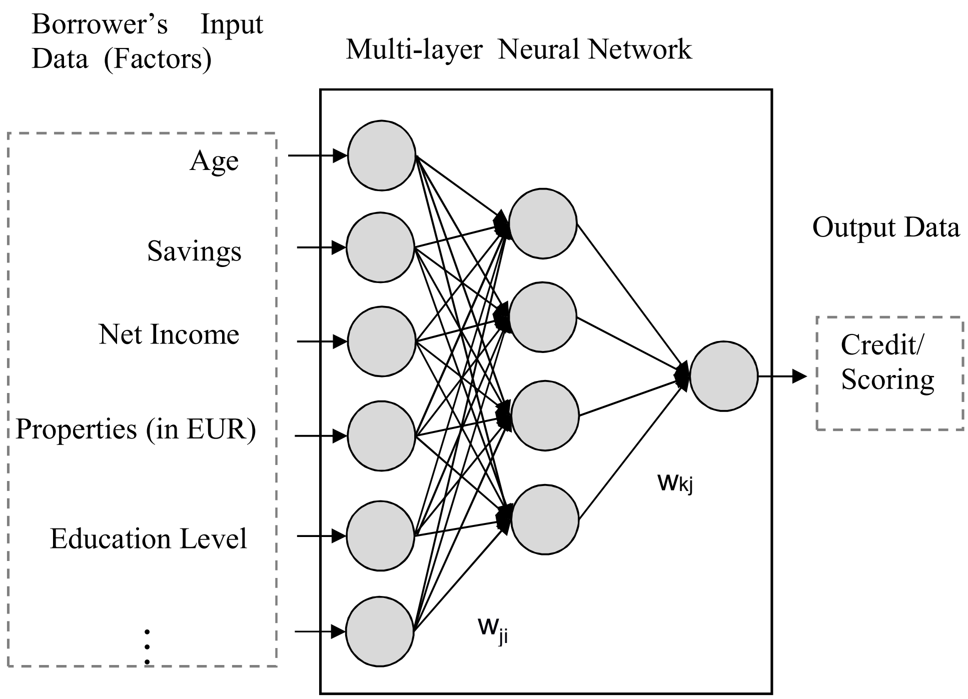


Fig. 2. Structure of a multi-layer neural network for the credit/scoring calculation

Aggregation within the structure

Aggregation of groups of input factors or partial evaluations lead to generalized evaluation and it can be a linear aggregation of weighted factors or a non-linear aggregation with saturation (see below Fig. 3 where a, b, c ... are weighted coefficients) as follows:

Linear: Aggregation = (a * Factor1 + b * Factor2 + ...+ N * Factor N) / (a + b + ... N)

Example: Aggregation= (3,5 * 63% + 4,5 * 36% + 7,2 * 48%) / (3,5 + 4,5 + 7,2) = 47,9%

Saturation: Aggregation = 1 - (1 – Factor 1) * (1 – Factor 2) * ... * (1 – Factor N)

Example: Aggregation = 100% - (100% - 63%) * (100% - 36%) * (100% - 48%) = 87,7%

The aggregations are yet again calibrated by 100% and do not exceed 100%.

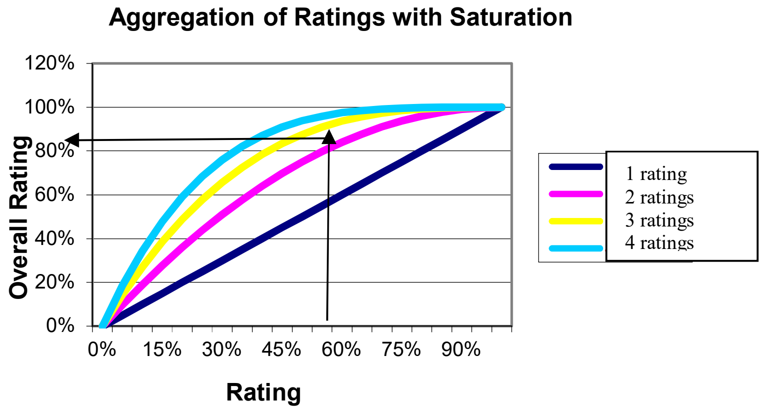


Fig. 3. Aggregation of ratings with saturation

Generation of results using neural networks

The training of the neural network is a complicated iterative process of adjusting the weighted matrices *Wji* and *Wkj* (see Fig. 2). Each element within the neural network works in two steps:

Calculation of the weighted input sum:

$$net_j = \sum_{i=1}^{A} w_{ji} x_i \tag{1}$$

and

Calculation of the activation function output:

$$h_j = f(net_j) \tag{2}$$

where f() is a non-linear activation function the neural network nodes. Here, the activation function is a bipolar sigmoid, similar to the *Z* function (see above):

$$f(net_j) = -1 + \frac{2}{1+e^{-net_j}} = \frac{1-e^{-net_j}}{1+e^{-net_j}} \tag{3}$$

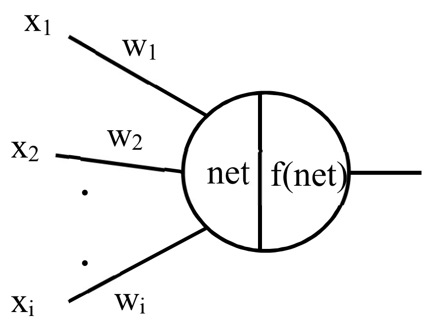


Fig. 4. Computing element (neuron) within the neural network

The propagation of the input values through the neural network and the generation of outputs are done in the following steps represented in matrix form:

1) Calculation of the matrix of sums of weighted input values for the elements from the hidden layer:

$$N_{ji} = W_{ji} X \tag{4}$$

2) Calculating the matrix of activation function values of the processing elements from the hidden layer:

$$H_{ji} = f\left(N_{ji}\right) \tag{5}$$

3) Calculating the matrix of sums of weighted input values for the elements from the output layer:

$$N_{kj} = W_{kj} H_{ji} \tag{6}$$

4) Calculating the activation function values of the processing elements from the output layer:

$$S_{kj} = f\left(N_{kj}\right) \tag{7}$$

In our case the output layer consists of only one element which produces the scoring value.

Training with all training exapmles

A training example for the neural network is a data record consisting of the borrowers' data with known scoring value. The scoring result is considered to be the wright one and must have been correctly calculated before that. The neural network model can be considered a complex mathematical function with many variables, which is not defined before the training. This function is concretized in the training phase and can be used in the following step to generate results when specific arguments (data of new borrowers whose scoring is to be calculated) are submitted to the function. For this reason, it is important the training to be carried out as precisely as possible.

For experimental check of the model for validation and scoring generation, a prototype with a neural network has been developed and its results for all training examples are shown in Fig. 5.

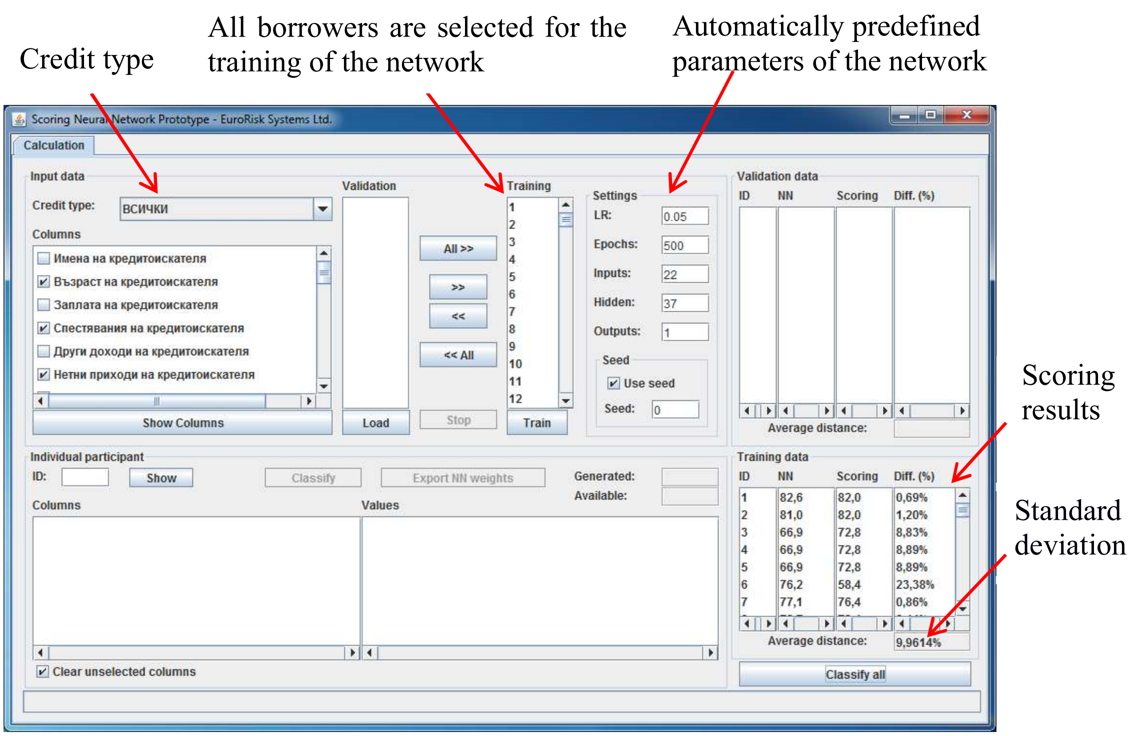
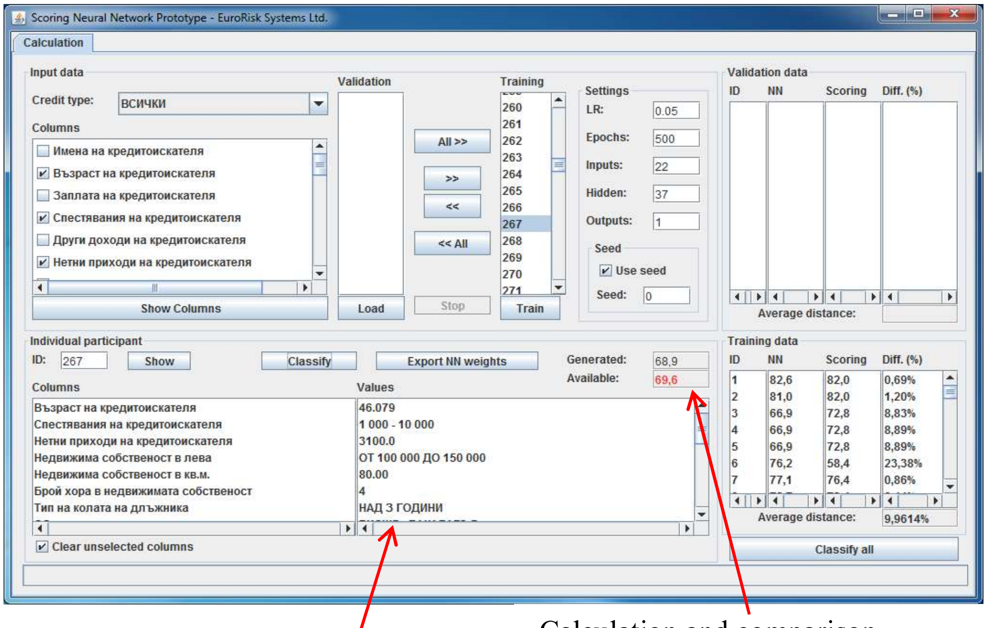


Fig. 5. Training using individual borrower´s data



Individual borrower's data

Calculation and comparison

of the borrower's scoring results

Fig. 6. Scoring results of an individual borrower

Training results when using a validating subset

Before the training phase, some of the presented examples are separated as a validation set and they do not participate in the training phase. The validation set is used to check the model results for new, unknown data. The results for the training and the validating sets are shown in Fig. 7. The average distance for the validation set is 15.14%, which is slightly more than the distance for the training set that is 11.02%, because the training set is already known to the model.

Selected data for validation Training set Results of the validation set

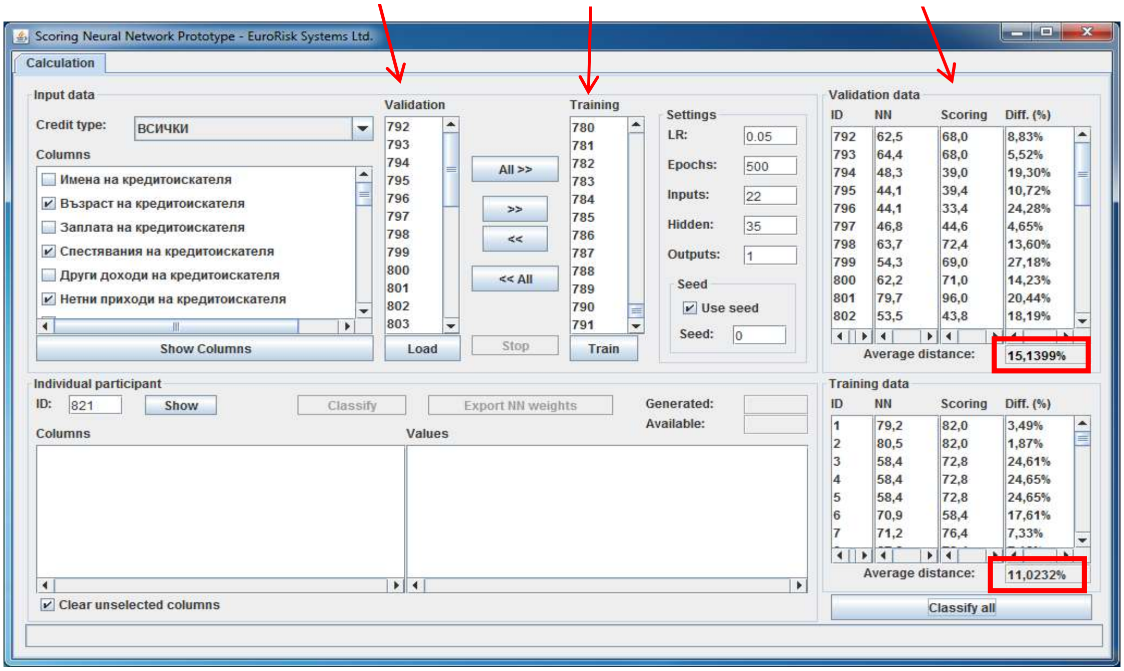


Fig. 7. Scoring results of the training and validating set

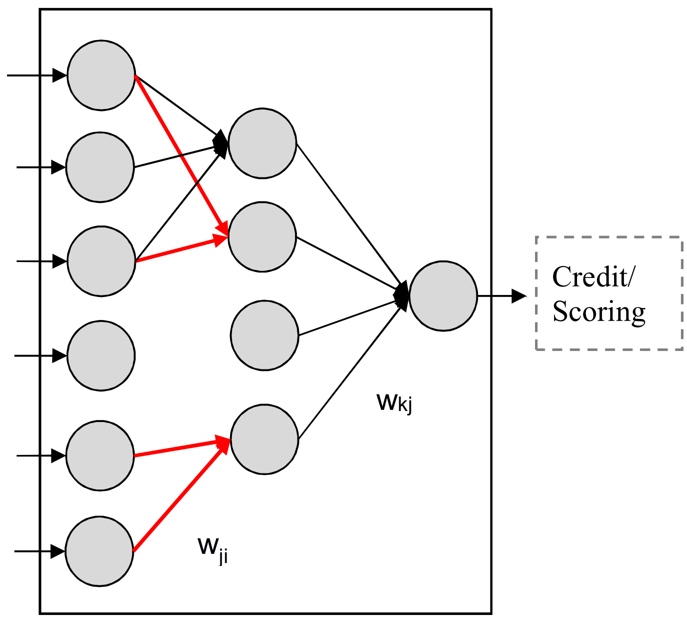
3. Analysis of the model

Analysis and reduction of the model structure

After the neural network training, the weights of the connections are available. They are used to analyze the network and to perform individual calculations with an arbitrary input vector, representing the borrower's data. To simplify the model, the connections with low value weights can be removed, thus allowing the determination of the factors significance and the reduction of the number of neurons. As a result, it is possible to overall eliminate insignificant factors or internal neurons. This clarifies the neural network and allows not only an easier interpretation and understanding of the model but also an identification of encoded dependencies in the weights matrix. This enables the elimination of insignificant factors or internal neurons, resulting in the network becoming clearer and the model easier to interpret and understand. It is also possible to find dependencies encoded in the weights matrix. Some nodes within the neural network (the rows of the weighting matrix) "learn" certain concepts and these concepts could be found and interpreted by such a reduction. Furthermore, particular factors (the columns of the weights matrix) influence the determination of the scoring values stronger and thus, after the reduction, stand out from other factors. The possibilities mentioned above affect the association of this analysis to the group of methods for knowledge finding (Data Mining). In the experimental research, the reduction is performed only for the weights matrix between the input and the hidden layer. In Fig. 8, a simplified structure of the neural network is shown, after removing the insignificant connections. The following peculiarities can be observed:

- The first, second and fourth neuron of the hidden layer are now dependent on perspicuous input factors. The fourth input neuron is independent of the other neurons. The neurons in the hidden layer represent specific intermediate concepts in the subject area, with a clear interpretation of the input factors. If the input factors "salary", "rent" and "other income" refer to the same neuron in the hidden layer, one can logically conclude that this neuron represents a summary of the term "income".

- The fourth input factor and the third hidden neuron can be removed, as they exert no influence.



Graph. 8. Reduced structure of the neural network

To reduce the connections within the weights matrix, a threshold value t of the standard deviation e_{ave} is set in advance, for all training examples M, between the scoring S of the reduced weighting matrix Wji and the scoring R at the non-reduced weighting matrix Wji. A linear search is performed for value r so that e_{ave} is not higher than value t.

$$e_{ave} = \sqrt{\frac{1}{M-1}\sum_{m=1}^{M}(D_m - \bar{D}_m)^2} \tag{8}$$

$$D_m = \frac{(S_m - R_m)}{S_m} \tag{9}$$

where S_m is calculated according to (7) for the m-th borrower and R_m is calculated in the same way but with a reduced matrix W_{ji} according to (4) as follows:

$$w_{ji} = \begin{cases} w_{ji}, \text{ако } w_{ji} < r \\ 0, \text{ако } w_{ji} \geq r \end{cases} \tag{10}$$

The results of the search of r are presented in Fig. 9, where a part of the reduced weights matrix is shown (the complete matrix contains 22 factors x 37 neurons in the hidden layer). The following parameters and results are displayed below:

Limit	r (the lowest weight coefficient for the reduction)
Scoring 0	Scoring without the reduction of the weights matrix
Error	t (maximal standard deviation between S and R)
Delta	Step for linear search
Fill	The filling of the matrix (percentage of unreduced connections to all connections from the weights matrix)

Intermediate Nodes	Age	Savings	Net Income	Properties in euros	Properties in qm	Number of persons in the household	Class of vehicle	Education level	Maritall status	Employer	Business
unit 27											
unit 28										0,231	0,296
unit 29			-0,229								
unit 30											
unit 31			-0,270								
unit 32	0,631	0,844	0,803	1,084	-0,793		-0,254			-0,335	-0,851
unit 33											
unit 34											
unit 35										-0,216	
unit 36											

unit 37											

NN Results

Scoring	Min	Max	Scale	Shift
69,058	14	96	0,02195	-1,20732

Weights

Limit	Scoring0	Diff	Error	Delta	Fill
21,00%	69,0583		10,00%	1,00%	7,99%

Fig. 9. Reduction of the weighted matrix

After the reduction of weights, it becomes evident that some neurons from the hidden layer (e.g. Unit 32) are more active in respect of the input information. Other factors (e.g. Unit 30, Unit 33, etc.) do not affect the scoring value and can thus be removed. Similarly, some factors stand out in relation to others, which emphasizes their greater influence on the scoring value. Factor „borrower's net income", for example, has a significant impact on the scoring value, whereas factors „number of persons in the household" or „borrower's marital status" have much less influence over the entire network. Fig. 10 displays a part of the non-reduced weights matrix.

Intermediate Nodes	Age	Savings	Net Income	Properties in euros	Properties in qm	Number of persons within the household	Class of vehicle	Education Level	Marital status	Employer	Business
unit 28	-0,009	-0,164	0,126	-0,058	0,115	-0,001	0,077	0,081	0,148	0,231	0,296
unit 29	0,119	0,050	-0,229	0,033	-0,125	0,018	-0,177	-0,152	-0,141	-0,049	-0,149
unit 30	-0,203	-0,042	0,142	0,066	0,073	-0,032	0,073	0,072	0,095	0,104	0,136
unit 31	0,124	-0,075	-0,270	-0,111	-0,183	0,080	-0,138	-0,012	0,021	-0,121	-0,100
unit 32	0,631	0,844	0,803	1,084	-0,793	0,206	-0,254	0,052	-0,110	-0,335	-0,851
unit 33	-0,035	0,036	0,149	-0,028	0,060	-0,070	0,068	0,000	-0,018	0,129	0,149
unit 34	-0,090	0,088	0,085	-0,046	-0,070	0,005	0,060	-0,013	0,044	-0,110	-0,065
unit 35	0,057	0,006	-0,127	-0,027	-0,140	0,081	-0,053	-0,028	-0,067	-0,216	-0,139
unit 36	-0,058	-0,008	-0,079	-0,053	-0,150	-0,001	0,034	-0,085	-0,056	-0,138	-0,122
unit 37	0,004	0,014	-0,004	0,058	-0,060	-0,035	0,075	-0,028	0,026	0,014	0,047

Fig. 10. Unreduced weighted matrix

The effects of the reduction of the network structure can be summarized as follows:

- Reduction of the model complexity;
- Determination of the factors' significance and influence;
- Optimization – removal of insignificant factors or hidden neurons;
- "Clarification" of the network, a simpler interpretation and a better understanding of the model;
- Identification of dependencies encoded in the weights matrix (Data Mining).

Contribution and sensitivity of factors

With the given weights matrix by which scoring values are generated, the sensitivity of scoring results in relation to each factor is determined, e.g. the extent to which changes of factors affect the scoring result.

$$c_i = \frac{\partial S}{\partial f_i} \tag{11}$$

Furthermore, the independence of factors can be determined by comparing the sum of sensitivities of each factor (c_b) to the sensitivity calculated by a simultaneous change of all factors (c_a).

$$c_a = \frac{\partial^n S}{\partial f_1 \partial f_2 \dots \partial f_n} \text{ and comparison with } c_b = \Sigma \frac{\partial S}{\partial f_i} \tag{12}$$

The determination of a factor's independence is based on Taylor series, with all higher or mixed differentials being zero. If the factors are independent, the sensitivities can be interpreted as linear contributions of factors to the scoring result.

To execute calculations, a change Δc is specified in advance, and all values w_{ji} corresponding to the factor *i* in the column of the weights matrix *Wji* (weights of all the links connected to an input neuron *i*) change as follows:

$$w_{ji} = w_{ji}(1 - \Delta c_i) \tag{13}$$

After that, the scoring value *S,* according to (7), is calculated.

Examples of factors	Sensitivity
Age of the borrower	-0,33%
Savings	0,68%
Net income	13,58%
Properties in euros	-0,46%
Properties in qm	4,07%
Number of persons in the property	1,11%
Vehicle class of the debtor	-0,15%
Education level	0,73%
Marital status	0,69%
Employer	5,57%
Business	-0,02%
Position	-0,20%
Sources of income	-1,06%
Work experience in years	3,91%
Position at the bank	2,31%
Area	0,18%
Target group of the bank	0,00%
Credit type	1,47%
Credit amount	6,79%
Interest income	-15,72%
Credit currency	-0,28%
Repayment period in months	-5,82%
Sum of sensitivities	**17,045%**
Sensitivity with simultaneous change	**17,058%**
Difference, independence and linearity	**0,0013%**

Table 1. Sensitivities of the scoring value to factors

The sensitivity c_i is obtained as a percentage difference of the calculated *S* and the scoring value *S0* calculated without change of the factor *i*. In the same way, the scoring *S* is calculated with a simultaneous change of all factors and the percentage difference is then again determined against *S0*. Results are shown in Table 1.

Values from Table 1 are calculated with Δci = 1% for each factor, where the effect on the scoring value, compared to the factor's unchanged value, is expressed as %. Sensitivities to scoring values are calculated as the sum of each change in a factor, as well as simultaneous changes of all factors. Results from Table 1 show that the total value of changes is approximately equal to the value of simultaneous changes. This determines zero influences of all higher and mixed changes, and this means independence and linearity of factors.

The combined effect of the calculation of sensitivities to factors can be represented as follows:

- Determination of the importance of factors by their changes = risk factor of the scoring value;
- Determination of risk in case of some simultaneous changes = risk of the scoring value;
- Check for factors independence;
- Determination of linearity and contribution of factors.

4. Future Developments

The aim of the represented methodology is to analyze and validate systems for the calculation of credit scorings. Interested credit institutions calculate scoring and rating values via these credit scoring systems. The analysis and validation are based on an available sample of scoring examples used to train the neural network as an analytical model. The analysis of this model can be further developed in the following directions:

- Evolution of the scoring model:
  Training the neural network with new examples that evolve and change over time, which causes structural changes, as well as changes in weighted coefficients. Of interest here is the automatic extraction of changes within the network, which is carried out via changes in the logic and the weights of the scoring system.

- Automatic calibration of the input factors at a given range, using possible values, e.g. the manager experience = (excellent, good, mediocre, etc.)

- If scoring examples are regularly present on the time axis, e.g. regular scoring calculations of the debtor (e.g. every 3 months), the following analysis is possible:

  - Stationarity test: permanent statistics on the time axis, e.g. expected value, trend and variation;
  - Seasonality test: repeating changes in the input data and in the rating system's behavior (e.g. in the winter months or during political events). Seasonality differs from cyclicality as the latter could reflect changes with different durations. Seasonality is predictable.

o Correlation test between factors, as well as correlation to other economical factors: the correlation between two series is calculated as follows:

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{[\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2]}} \tag{14}$$

o Autocorrelation test of factors = correlation between points from one series, that are located at the same distances from one another. For a given number of N observations, $x_1,...,x_N$, N-1 observation pairs (x_1, x_2), (x_2, x_3), ...,(x_{N-1}, x_N) can be formed. If the first values of each pair are considered as one variable, and the second values as a second variable, the correlation coefficient can be calculated between x_t and x_{t+1}:

$$r_1 = \frac{\sum_{t=1}^{N-1}(x_t - \bar{x}_{(1)})(x_{t+1} - \bar{x}_{(2)})}{\sqrt{\left[\sum_{t=1}^{N-1}(x_t - \bar{x}_{(1)})^2 \sum_{t=1}^{N-1}(x_{t+1} - \bar{x}_{(2)})^2\right]}} \tag{15}$$

In the same way, it is possible to calculate the correlations between observations that are offset from one another at a distance k:

$$r_k = \frac{\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{N}(x_t - \bar{x})^2} \tag{16}$$

This is called an autocorrelation coefficient with a lag k. For smaller N, the formula below is more precise:

$$c_k = \frac{1}{N-k}\sum_{t=1}^{N-k}(x_t - \bar{x})(x_{t+k} - \bar{x}) \tag{17}$$