

MARKET RISK EVALUATION USING MONTE CARLO SIMULATION – METHODOLOGY & FEATURES

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1. Introduction

Market Risk involves the uncertainty of future earnings, resulting from changes in various independent underlying assets within the market environment (prices of assets, interest rates, FX rates, Spreads, etc.) for a particular future time point (risk horizon). The market risk of a position or a portfolio is measured by a single value, called market Value at Risk (VaR). Market VaR represents unexpected losses at adverse move. It is a measure of risk, based on the probability of loss at a specific time horizon, in which this loss can be expected to occur. Bank regulators use VaR to set capital requirements for bank trading accounts, as VaR models can be used to estimate capital losses due to market risk.

The PMS includes three methods for Market Risk calculation: parametric VaR/CoVaR, historical simulation and structured Monte Carlo simulation, which is the most powerful one.

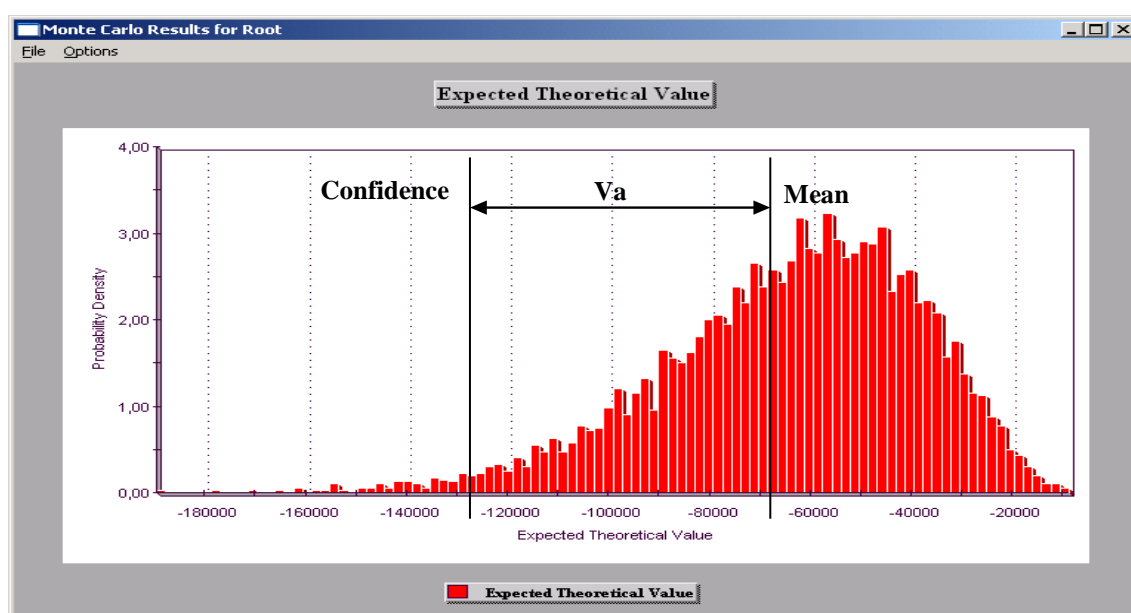


Fig. 1 Monte Carlo Simulation and VaR of a short Swaption

A structured Monte Carlo simulation engine in the PMS produces price distributions of a single financial position or portfolio. The aggregation is performed by applying numerical calculation procedures on a large set of possible market scenarios, that are derived from statistic data for market moves from historical series. The structured Monte Carlo simulation produces better VaR results for non-linear instruments, such as options, long-term cash flow assets and pay off of structured products, retaining the non-linear nature of the assets (s. Fig. 1).

2. Structured Monte Carlo Simulation

2.1 Evaluation Methodology

The Monte Carlo module for market risk (s. Fig. 2) is based on the following methodology:

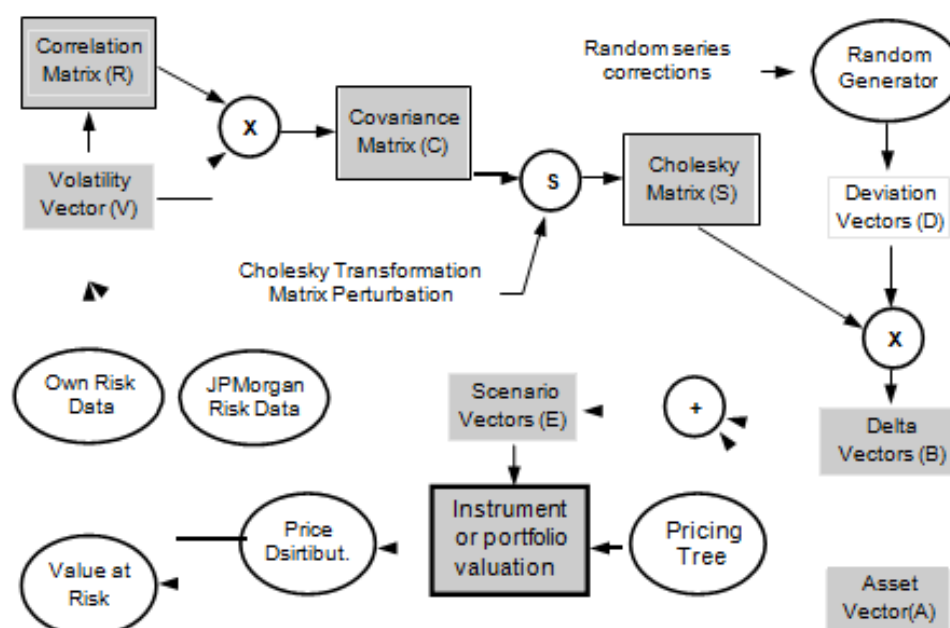


Fig. 2 Structured Monte Carlo Simulation

1. Volatility vector (**V**) and correlation matrix (**R**), for a set of market risk drivers, such as interest rates, FX rates, prices and spreads, are produced from statistics of their own historical series in the PMS data base, or are imported from JPMorgan standard risk data sets.
2. The covariance matrix (**C**) is obtained by multiplying the volatility vector (**V**) with the correlation matrix (**R**).
3. The Cholesky matrix **S** is constructed from the covariance matrix (**C**), so that

$$\mathbf{S} \times \mathbf{S}' = \mathbf{C},$$

where **S'** is the transposed matrix.

4. Matrix Perturbation adjusts the covariance matrix to be positive definite, in case the original matrix is not positive definite, as a result of dependencies from risk drivers.
5. A random generator produces independent samples for each market driver, using standard normal distribution SND (0,1) stored into deviation vectors (**D**). A SVD correction and a normal form correction, applied to random vectors, ensure normal distribution, where; mean = 0, standard deviation = 1, skewness = 0, kurtosis = 0, and correlation between factors = 0. The high quality of the random series reduces the number of Monte Carlo runs. 5.000 to 10.000 runs is usually enough to obtain good results.
6. Deviation vectors (**D**) are multiplied by the Cholesky matrix (**S**), taking into account the covariance between every risk driver pair. This produces delta vectors (**B**) of normally distributed and correlated deviations.
7. Vectors (**B**) are applied to actual risk driver values (interest rates, FX rates, spreads, etc.) in the asset vectors (**A**) and thus produce scenario vectors (**E**) for Monte Carlo runs. These scenario vectors then compute the pricing expression value using pricing trees, in order to represent instruments and portfolios supplied by a Pricing Tree Generator. Pricing trees are specialized data structures that ensure high performance of the simulation.
8. Steps 6 is repeated a number of times (5000...10000). Large scenario set cause the price distribution calculation of pricing expressions. The resulting price distribution is constructed by counting the number of values within many adjacent small ranges, after which the Value at Risk is calculated by a numerical integration of the distribution density, using a confidence percentage, for example 1% for the upper limit of the price distribution.

2.2 Monte Carlo Simulation Mathematics

2.2.1 Calculation of the Covariance Matrix

$$C_{ij} = R_{i,j} * V_i * V_j$$

where:

$i = 1 \dots n$ (matrix width),

$j \leq i$ (triangle matrix)

where C_{ij} - Element from covariance matrix

R_{ij} - Element from correlation matrix ($R_{ii}=1$)

V_i, V_j - Element from volatility vector

2.2.2 Construction the Cholesky Matrix S

$$\mathbf{S} \times \mathbf{S}' = \mathbf{C},$$

where

\mathbf{S}' – transposed matrix

a_{ij} – elements of the matrix S

For example, for a matrix width = 3, the following matrix operation applies:

$$\mathbf{C} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \times \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ 0 & a_{22} & a_{32} \\ 0 & 0 & a_{33} \end{pmatrix}$$

The matrix multiplication involves formulas for the calculation of elements of the Cholesky matrix:

$$a_{ii} = \left(c_{ii} - \sum_{k=1}^{i-1} a_{ik}^2 \right)^{1/2}$$

$$a_{ij} = \frac{1}{a_{jj}} \left(c_{ij} - \sum_{k=1}^{i-1} a_{ik} \cdot a_{jk} \right) \quad j = i + 1, i + 2, \dots, N$$

2.2.3 Matrix Perturbation

This is a method for transforming a non-positive definite matrix into a positive definite one. A matrix is said to be symmetric if $\mathbf{A} = \mathbf{A}'$, where \mathbf{A}' is the transpose of \mathbf{A} . Therefore, covariance and correlation matrices are real and symmetric. A real symmetric matrix \mathbf{A} is said to be positive definite if $\mathbf{x}'\mathbf{A}\mathbf{x}$ for every vector \mathbf{x} different from 0. Given this definition, one can prove

that a real symmetric matrix is positive definite if and only if all its eigenvalues are greater than zero.

The question is: “How do the eigenvalues of a matrix **A** change if it is subject to a perturbation **A → A+E**?” Because the eigenvalues are continuous functions of the entries of **A**, it is natural to think that, if the perturbation matrix **E** is small enough, the eigenvalues should not change too much. It would be undesirable for our purposes to drastically change a matrix in order to turn it into a proper covariance matrix.

The Monte Carlo engine of PMS uses the following perturbation:

A → A+e(I-A) = B, where **e** is a scalar and **I** is the identity matrix.

If **e** = 0, then **A → A**; and if **e** = 1, then **A → I** which is a clearly positive definite matrix. Therefore, since the eigenvalues of a matrix are continuous functions of their entries, there is an infinite number of scalars **e** in the interval [0, 1] that, given by the equation above, are a positive definite matrix. The task is to obtain the min of **e**, which is calculated from min of the eigenvalues.

2.2.4 Generation of standard normal distribution SND (0,1) vector

$$V_1 = 2 * Rnd_1 - 1 \quad V_2 = 2 * Rnd_2 - 1 \quad S = V_1^2 + V_2^2$$

for $S \geq 0$:

$$SND_1 = V_1 * \sqrt{\frac{-\log(S)}{S}} \quad SND_2 = V_2 * \sqrt{\frac{-\log(S)}{S}}$$

where:

Rnd1, Rnd2 – two adjacent random numbers (0..1)

SND1, SND2 – two adjacent random numbers with standard normal distribution.

2.2.5 Correction of random series

2.2.5.1 SVD Correction of expected value, standard deviation and correlation

The first correction of expected value, standard deviation and correlation is created by common post-processing. The approach for the correction is based on the calculation of statistical figures of random numbers and the application of differences to the ideal case.

Formal task

Given the matrix $\mathbf{X}(\mathbf{n}, \mathbf{k})$, where n lines contains the generated \mathbf{k} -dimensional normally distributed random samples. The statistical parameters of these sample can be calculated: $\mathbf{m}(\mathbf{k})$ vector of mean values and $\mathbf{K}(\mathbf{k}, \mathbf{k})$ covariance matrix. The distribution caused by the Matrix \mathbf{X} follows the normal distribution $\mathbf{N}^{(k)}(\mathbf{m}, \mathbf{K})$. It also requires that the sample distribution of target $\mathbf{N}^{(k)}(\mathbf{v}, \mathbf{C})$ should have specific parameters; in this particular case, $\mathbf{v} = (0, 0, \dots, 0)$, where the matrix \mathbf{C} should be the unit matrix (overall 0 and everywhere on the main diagonal = 1). There are two formal challenges:

- The task of shifting the statistical means from point \mathbf{m} to point \mathbf{v} ;
- The task of transforming the statistical structure, represented by the covariance matrix \mathbf{K} , to the target matrix, represented by \mathbf{C} .

Solution of the task

The first challenge is easily resolved by shifting the statistical means with the difference vector \mathbf{r} .

The second challenge requires the conversion of statistical sample \mathbf{X} into a new \mathbf{Y} , which owns the target parameter. The solution is found in the form of a linear operator with the conversion matrix \mathbf{S} sought. Every vector \mathbf{Y}_i is determined in the following manner:

$$y_i = S^T \cdot x_i + r, \quad i = \overline{1, n}$$

Statistical parameters of the sample \mathbf{Y} are calculated as follows:

The vector of mean values:

$$\begin{aligned} E\{y_i\} &= v = \\ E\{S^T \cdot x + r\} &= r + E\{S^T \cdot x\} = \\ r + S^T \cdot E\{x\} &= r + S^T \cdot m \end{aligned}$$

The difference vector \mathbf{r} can be derived from the expression above:

$$r = v - S^T \cdot m$$

Elements of the covariance matrix are:

$$\begin{aligned}
 E\{(y_i - v) \cdot (y_i - v)^T\} &= C = \\
 &= E\{(S^T \cdot x_i + r - v) \cdot (S^T \cdot x_i + r - v)^T\} = \\
 &= E\{[S^T \cdot x_i + r - (r + S^T \cdot m)] \cdot [S^T \cdot x_i + r - (r + S^T \cdot m)]^T\} = \\
 &= E\{[S^T \cdot (x_i - m)] \cdot [S^T \cdot (x_i - m)]^T\} = \\
 &= S^T \cdot E\{(x_i - m) \cdot (x_i - m)^T\} \cdot S = \\
 &= S^T \cdot K \cdot S
 \end{aligned}$$

For the sustainability of the matrix **S**, it is necessary that matrices **K** and **C** are commutative. This condition is satisfied, as the covariance matrices are symmetrical. The correction, implemented for random series, uses a **SVD** algorithm (Singular Value Decomposition) in determining the conversion matrix **S**.

2.2.5.2 Correction for normal distribution forms

The purpose of the correction form is to correct figures of the second order (Kurtosis = 0, Skewness = 0), i.e. the distribution form of the ideal normal distribution should be reached, but the results of the first correction should be retained.

The following aspects are important in the implementation of this correction:

- the inverse function of a standard normal distribution SND (0.1) is known as a strict linear function, i.e. a synthetic linear numerical series could be produced after converting an ideal standard normal distribution.
- The necessary correlation between random series is 0 and is given by the sequence of random numbers and through their mutual order in the random series. The sequence of each random number in a random order should be preserved, with regard to the sequence of all other random numbers in other random series .

The points above indicate follows an approach to the implementation of the correction, using the following steps:

- Each time series is converted by the inverse function.
- The converted random numbers should be equally distributed, because of the original standard normal distribution. Of course, this is not the case due to the random generator and the task of correcting the form to an ideal equal distribution.

- The random numbers are then sorted, with their original order saved. The new order should follow a linear function. Again, this is not the case, as the numbers vary somewhat within the linear function.
- The figures are then overwritten by a synthetic linear function (from 0 to 1) and converted back again to normal distribution, thus the arise of random series with a very good quality.
- As the final step, the original order is restored, in order to reconstruct the initial series correlation.

2.2.6 Obtaining the delta formal distribution vector

$$\overline{B} = |S| \times \overline{D}$$

S – Cholesky matrix

D – standard normal distribution vector

B – delta normal distribution vector

2.2.7 Calculation of the scenario vector

The scenario vector is obtained by applying the delta normal distribution vector on the asset vector, that contains interest rates, foreign exchange rates, etc.

For interest rates:

$$A_i^{Scenario} = A_i * (1 + B_i)$$

For prices and foreign exchange rates:

$$A_i^{Scenario} = A_{t_i} \square e^{B_i}$$

where:

Bi – Scenario

Ai – Asset

2.2.8 Construction of the price distribution and calculation of Value at Risk

After an instrument or a portfolio is evaluated for all Monte Carlo runs (5000. 10000), using different scenarios, the price distribution is calculated by counting the number of portfolio values within small adjacent intervals around the zero scenario. Value at Risk is calculated by the numerical integration of the distribution density function, using a confidence level.

$$\text{Value at Risk} = \text{Portfolio_Mean} - \text{Confidence_Value}$$

$$\text{Confidence_Value} = \text{Portfolio Value for the confidence level}$$

2.3 Instrument Representation by Pricing Expressions

Large numbers of instruments – with closed pricing formulas, such as Loan, Bond, Interest Rate Swap, Cross Currency Swap, Interest Rate Future, FRA, Bond Future, Floating Rate Note (FRN), etc. – can be represented by cash flow series, including payments in different currencies. The prices of these series can be symbolically expressed via pricing expressions, that have a straight representation on pricing trees.

The aim is to obtain **fast simulation tree structures** for the pricing formulas, in order to achieve high performance of subsequent simulation runs.

2.3.1 Syntax of Pricing Expressions

Pricing expressions are represented by standard arithmetic expressions, using a set of simple syntax rules:

expression	==> (term ('+' '-') expression)
 term term	==> (factor ('*' '/') term)
factor	
factor	==> '(' expression ')'
	==> 'df' '(' currency ',' [market ','] period [',' number] ')'
	==> 'fx' '(' currency ')'
	==> 'ei' '(' currency ')'
	==> number
currency	==> 'usd' 'eur' 'gbp'
market	==> 'b' 's'
period	==> number

The meaning of the stochastic factors is the following:

df(gbp, s, 4m, 2500)	- price (discount factor) of 4 month 2500 gbp cash flow (swap market)
df(usd, b, 1.5y)	- price 18 months 1 usd cash flow (bond market)
fx(eur)	- foreign exchange rate leading currency for 1 eur
ei(eur)	- equity index for eur
b or s	- bond or swap market

2.3.2 Pricing Formula Examples

Presented below are pricing formula examples that can be used as templates for other instruments. Each example is commented using some abbreviations:

Fr - frequency; Ma - maturity; Ir - interest rate; Po - position;

Cu - currency; Na - notional amount; Fp - future price; Fx - FX rate

1. Government Bond: Fr=12m, Ma=2y, Ir=6,4%, Na=10000, Ppo=long, Cu=eur

$$\text{FixedCF(DEM)} = 10000 * (0.064 * (\text{DF}(\text{eur}, \text{b}, 12\text{m}) + \text{DF}(\text{eur}, \text{b}, 24\text{m})) + \text{DF}(\text{eur}, \text{b}, 2\text{y}))$$

simplified to: $\text{DF}(\text{eur}, \text{b}, 360, 640) + \text{DF}(\text{eur}, \text{b}, 720, 10640)$

2. Float cash flow: Fr=1y, Ma=2y, Ir=float, Na=1000, Po=long, Cu=usd

$$\text{FloatCF(USD)} = ((\text{df}(\text{usd}, \text{s}, 0\text{y})/\text{df}(\text{usd}, \text{s}, 1\text{y}) - 1) * \text{df}(\text{usd}, \text{s}, 1\text{y}) + (\text{df}(\text{usd}, \text{s}, 1\text{y})/\text{df}(\text{usd}, \text{s}, 2\text{y}) - 1) * \text{df}(\text{usd}, \text{s}, 2\text{y})) * 1000$$

simplified to: $\text{df}(\text{usd}, 0\text{y}, 1000) \rightarrow$ i.e. risk free value of 1000

3. Profit&Loss of 7x3m FRA: Ma=7m, Na=1000, Po=short, Cu=gbp, Ir=0.06

$$\text{P\&L-FRA(GBP)} = 8000 * (0.06/4 - \text{df}(\text{gbp}, 7\text{m})/\text{df}(\text{gbp}, 10\text{m}) - 1) * \text{df}(\text{gbp}, 10\text{m})$$

simplified to: $\text{df}(\text{gbp}, 10\text{m}, 8012) - \text{df}(\text{gbp}, 7\text{m}, 8000)$

4. Representation of a simple instrument with two payments IR(t+1) and IR(t+2)

Payments are in a foreign currency. Discount factors are split on standard zero curve vertices. The pricing expression is as follows:

$$Price = (IR_{t+1} * DF_{t+1} + IR_{t+2} * DF_{t+2}) * FX * Nominal$$

The corresponding pricing tree is shown in Fig 3.

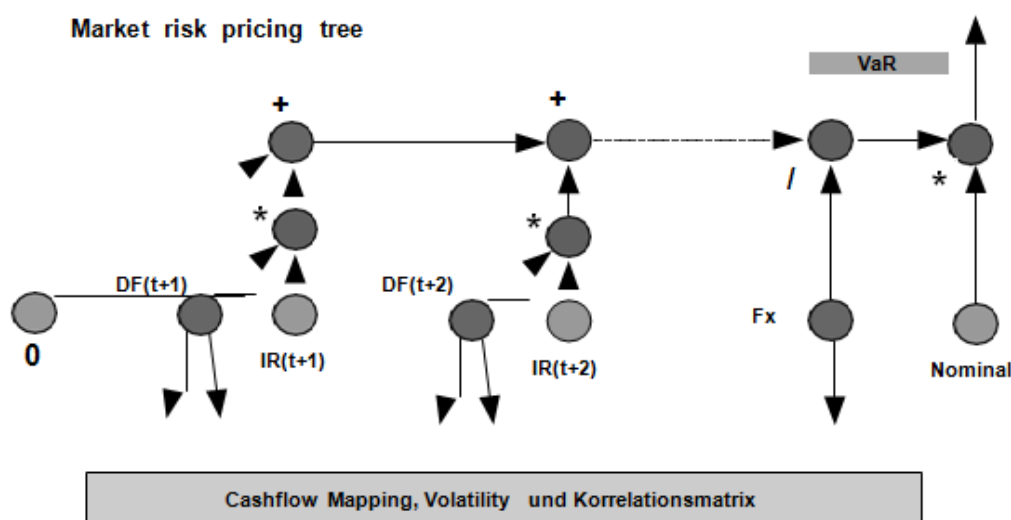


Fig. 3 Pricing Tree Example

2.3.3 Semantic of Pricing Expressions

2.3.3.1 Forward Pricing

The evaluation of pay offs requires forward pricing for various future evaluate dates, i.e. pricing formulas should be able to express forward prices and discount factors:

$$\text{forward_df}(t_e, T_i) = \text{df}(t_0, T_i) / \text{df}(t_0, t_e)$$

2.3.3.2 Forward Interest Rates

Forward interest rates should be expressed via discount factors from zero curves for the desired currency, using arithmetic operators:

$$\text{forward_interest_rate}(T_{i-1}, T_i) = (\text{df}(t_0, T_{i-1}) / \text{df}(t_0, T_i) - 1) / \text{Period}$$

2.3.3.3 FX Rate Representation

The representation of FX rates should follow a straight approach, based on the leading currency, which is the base currency of the FX-Table in use and is connected to volatility and correlation data of all FX rates related to this leading currency. Using the notation:

$fx(Cu1, Cu2)$ foreign exchange rate

$Cu1 \rightarrow Cu2$ (how many units of Cu1 costs one unit of Cu2),

We can express all needed rates assuming, for example, the USD as the leading currency:

$$fx(USD, Cu2) = fx(Cu2) \quad fx(Cu1, USD) = 1/fx(Cu1)$$

$$fx(Cu1, Cu2) = fx(Cu2) / fx(Cu1)$$

2.3.4 Representation of Instruments

Two concepts – cash flows (or pay offs) and option rights – are sufficient to explain the nature of most financial instruments, using closed formulas that can be simulated on a pricing tree.

2.3.4.1 Interest rate and FX instruments

Redemption payments on future dates are specified according to an amortizing schedule; the final redemption payment zeroes the initial debt. Fix or float interest rate payments on the remaining debt are defined according to future interest rate periods. Linear combinations (portfolio) of this basic schema are used to express more complex instruments, such as interest rate swap, reverse floater, etc., or instruments like the cross currency swap, involving two currencies. All instrument payments can be interpreted as one or several cash flow series, in one or more currencies, or as a portfolio of single payments in one or in more currencies. Therefore, the theoretical price of this instrument group is given by a linear sum of discounted payments, grouped according to their currency and aggregated to a consolidation currency via spot foreign exchanges.

$$Price = \sum_j fx_j * \sum_i npv_i, \quad \text{where } npv_i = cf_i * df_i$$

All cash flow payments cf_i are split and cumulated on standard zero curve vertices. This increases the Monte Carlo performance dramatically, as the number of simulated cash flow prices is reduced to the number of standard zero curve vertices. For example, let us assume that 1000 bonds are represented on a small set of vertices, such as 1, 3, 6, 12 months and 2, 3, 5, 7, 10, 30 years. From that, three basic nodes are derivable:

fix cash flow: $fix(currency, market_id, time, value)$

float cash flow: **float**(currency,market_id,time1, time2, value)

foreign exchange: **fx**(currency, market_id, time, value)

2.3.4.2 Stock instruments

Shares, including dividends, can be treated as cash flow, according to current market share price and future dividend payments. Abstract assets – such as stock index, used as the underlying of options or futures – can be seen as payments, too, as there is a fixed price for every stock index point. Share volatility is usually linked to stock index volatility by Beta factors, but shares can also be treated separately.

2.3.4.3 Future instruments

Future contracts grant the symmetric right and obligation to buy or to sell assets on a future date, for a predefined asset price. This deal type can be expressed via a linear combination (portfolio) of two lending/borrowing transactions: one on today's date and the other on a future date. In this way, futures can be fully explained by these two transactions. Interest rate futures, including FRAs, imply the lending/borrowing on two different future dates, while foreign exchange futures and outright deals deal with future exchanges of two currencies. Futures incomes (Profit & Loss) can be represented in the same way as interest rate asset instruments, i.e. by a linear sum of discounted payments in one or two currencies.

2.3.4.4 Option instruments

The representation of options is more complex and depends on contract conditions, as well as on the period of the option's right. The current version of the market risk engine in PMS only values European options on a generalized Black & Scholes node. Complex, path dependent multi-option rights are valued by linearization and are based on key rate values.

Generalized Black & Scholes Valuation

A basis node, that is incorporated into a pricing tree (s. Fig. 4) and that represents general European options –including many exotics, such as digital option – is defined as follows:

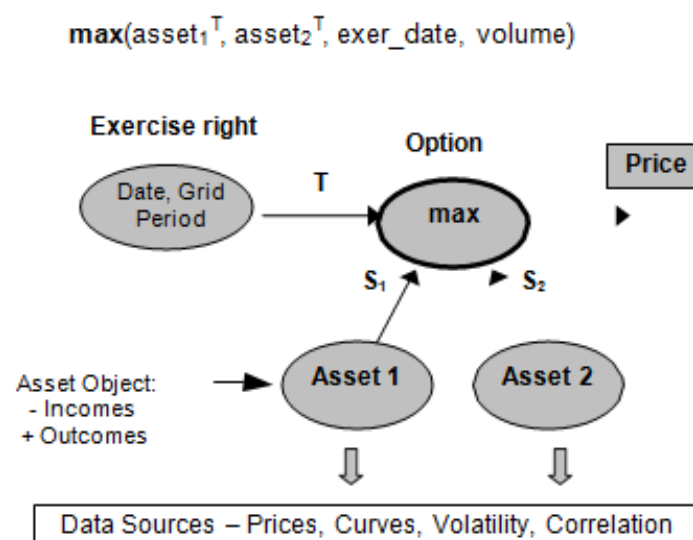


Fig. 4 Option Valuation within generalized Black & Scholes Node

The following data – price, annual volatility, implied term structure constructs, currency, asset incomes and outcomes, and market_id that enables the linking to market data sources – are properties of the underlying assets, not the option self, i.e. they should be supplied by the underlying asset or through an underlying asset object (a more complex asset).

3. Dimensions and Risk Measures of Monte Carlo Simulations

3.1 Risk Drivers

The following risk drivers are recognized for every currency and for every market, and are fully correlated (s. Fig. 5):

- Interest rate curves, represented by standard vertices in money and capital market segments
- FX rates
- Shares and stock indexes
- Spread curves that simulate credit risk, based on CDS Spreads
- Spread indices, e.g. ITRAX, that simulate credit risk, based on CDS spread for Index baskets

Vola and Correlation Matrix													
Scenario <input type="text"/> <input type="button" value="Reset"/>													
<input type="button" value="Store"/> <input type="button" value="Ok"/> <input type="button" value="Cancel"/>													
Diagonal: Vola (=sigma) Otherwise: Correlations													
08 Swap K	2009 K-Governm DKK	2009 DKK-Swap DKK	2010 K-Governm DKK	2010 DKK-Swap DKK	CDI CDS-Index 5Y EUR	581365 M. Label-New EUR	67573 Extra Incor EUR	70077 gage Bond EUR	74498 GLOBAL PR. EUR	74690 EUR.MIT.TH EUR	74691 2 GLOB BD. EUR	74694 LOBAL GROV EUR	75470 DYNAMIC EC EUR
0,0078	0,0078	0,9903	0,7603	0,9930	0,0078	-0,3223	0,0002	0,1293	-0,0474	-0,0429	0,0013	-0,0703	-0,0013
0,8076	0,3151 %	0,8033	0,9915	0,8018	0,0349	-0,3893	-0,0006	0,0911	-0,0310	-0,0399	0,0257	-0,0064	-0,0707
0,9963	0,8033	0,2557 %	0,7842	0,9970	0,0732	-0,3066	0,0258	0,1271	-0,0390	-0,0376	0,0758	-0,0766	-0,0749
0,7885	0,9915	0,7842	0,3474 %	0,7823	0,0248	-0,3791	-0,0049	0,0983	-0,0094	-0,0504	0,0276	-0,0172	-0,0567
0,9930	0,8018	0,9970	0,7823	0,2811 %	0,0755	-0,2959	0,0268	0,1268	-0,0360	-0,0237	0,0721	-0,0708	-0,0707
0,0670	0,0349	0,0732	0,0248	0,0755	2,5756 %	0,0690	-0,0891	-0,0691	0,0417	0,0362	-0,0479	0,0825	0,0246
0,3229	-0,3893	-0,3066	-0,3791	-0,2959	0,0690	1,0304 %	0,1426	-0,0749	0,0541	0,1120	-0,1022	0,2547	0,1046
0,0302	-0,0006	0,0258	-0,0049	0,0268	-0,0891	0,1426	0,4217 %	0,0955	0,0943	0,0879	-0,1057	0,0272	0,1490
0,1295	0,0911	0,1271	0,0983	0,1268	-0,0691	-0,0749	0,0955	0,3839 %	0,1198	-0,0135	0,0513	-0,0769	0,1741
0,0474	-0,0310	-0,0390	-0,0094	-0,0360	0,0417	0,0541	0,0943	0,1198	1,0718 %	0,5834	-0,3302	0,4284	0,8166
0,0429	-0,0399	-0,0376	-0,0504	-0,0237	0,0362	0,1120	0,0879	-0,0135	0,5834	0,9121 %	-0,4756	0,5838	0,5990
0,0813	0,0257	0,0758	0,0276	0,0721	-0,0479	-0,1022	-0,1057	0,0513	-0,3302	-0,4756	0,2732 %	-0,3051	-0,3446
0,0785	-0,0064	-0,0766	-0,0172	-0,0708	0,0825	0,2547	0,0272	-0,0769	0,4284	-0,3051	1,0371 %	0,5067	0,5067
0,0813	-0,0707	-0,0749	-0,0567	-0,0707	0,0246	0,1046	0,1490	0,1741	0,8166	0,5990	-0,3446	0,5067	0,6185 %
0,0472	-0,0584	-0,0384	-0,0548	-0,0242	0,0269	0,1009	0,0010	-0,0070	0,0893	0,1171	0,1367	0,1140	0,1137
0,0725	-0,1126	-0,0614	-0,1091	-0,0488	0,0676	0,1267	-0,0136	0,0624	0,1492	0,0744	0,2268	0,1332	0,1609

Fig. 5 Correlation Matrix and Volatility of Risk Drivers

3.2 Data Sets

The following market risk data can be used for Monte Carlo Simulations:

- Standard JPMorgan data sets: daily, monthly and basel downloaded from the RMG site.
- Own data sets calculated from time series and stored in the PMS data base (s. Fig. 6)

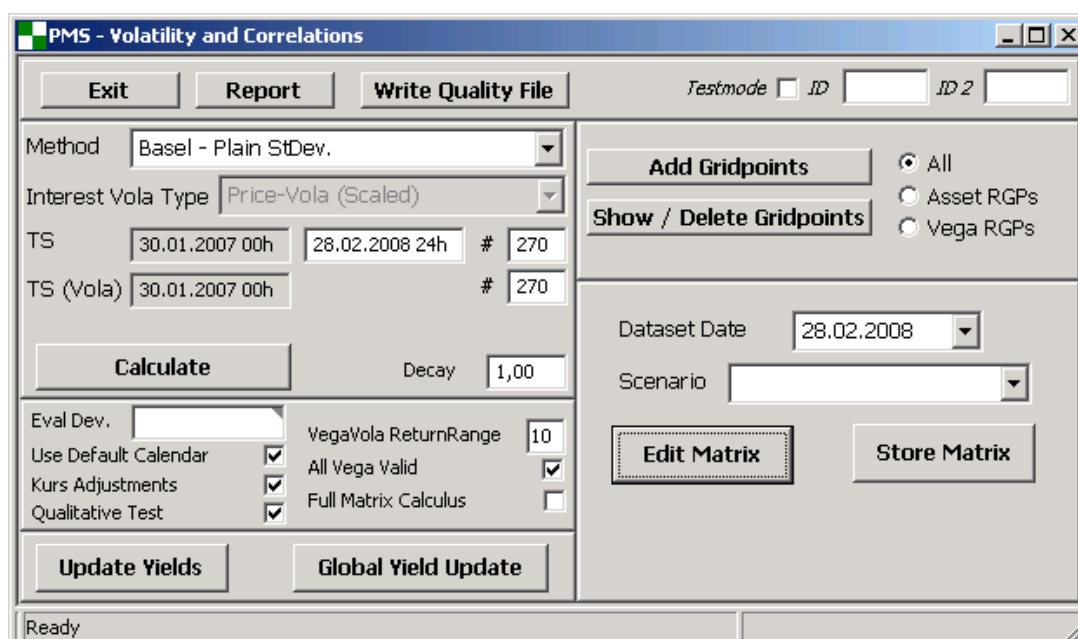


Fig. 6 Calculation of own Data Series (Correlation Matrix and Volatility)

3.3 Risk Types

The following market risk types are calculated and presented, separately as well as together, in the Monte Carlo simulation (s. Fig. 7):

- Only FX Risk
- Only Share and Stock Index Risk
- Only Interest Rate Risk
- Only Spread and spread Index Risk
- Total Risk
- Incremental Risk
- Marginal Risk
- VaR Standard and Expected Shortfall at Confidence

Incremental VaR

A position's Incremental VaR (IVaR), with respect to a portfolio, can be thought of as the amount of risk that the position is adding to the portfolio (s. Fig. 8). In other words, Incremental VaR tells us how the VaR of our portfolio would change if a specific position were to be sold. Incremental VaR can be formally defined as the difference between the VaR of a total portfolio and the portfolio's VaR without the position in question.

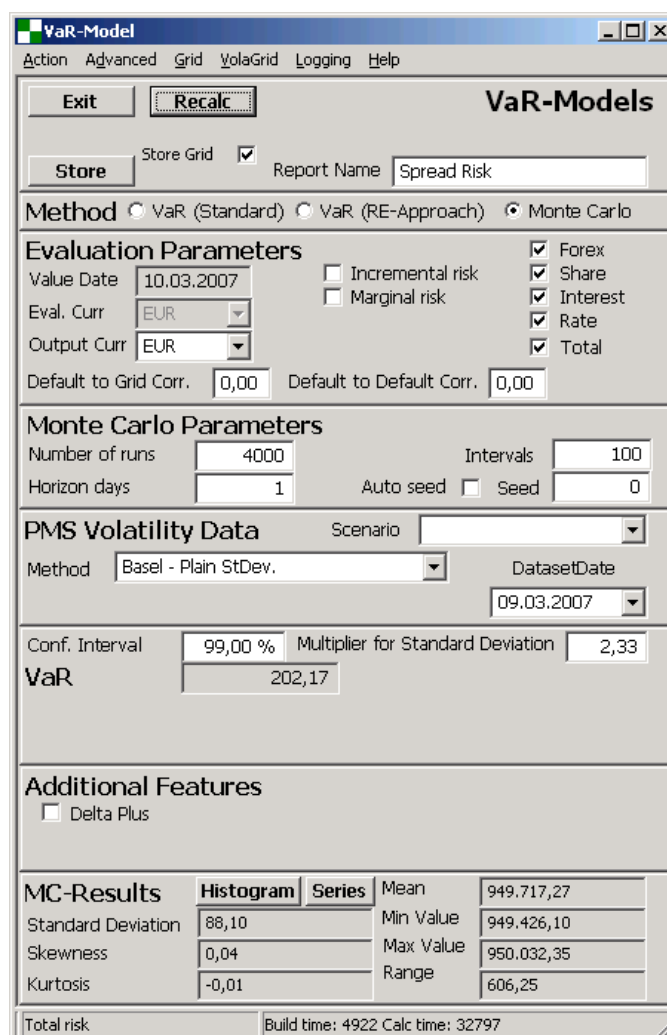
Marginal VaR

Marginal VaR (MVaR) expresses the potential effect that buying or selling a relatively small portion of a position, for instance 1 %, has on the overall risk (s. Fig. 8). For example, in the

process of rebalancing a portfolio, we often wish to decrease our holdings by a small amount, rather than liquidate the entire position. Marginal VaR (MVaR) represents statistic data that provides information regarding the sensitivity of VaR to changes in portfolio holdings.

Expected Shortfall

Expected Shortfall (VaR-ES) is a subadditive risk statistic that describes how large losses behave on average, when they exceed the VaR level. This data hence provides further information about the tail of the P&L distribution. Mathematically, we can define Expected Shortfall as the conditional expectation of portfolio losses, given that they are greater than the VaR. Expected Shortfall is a useful statistic that provides valuable additional information and is important as a measure of fat-tailed distributions.



VaR-Model

Action Advanced Grid VolaGrid Logging Help

Exit Recalc

Store Store Grid ☒ Report Name Spread Risk

Method ☐ VaR (Standard) ☐ VaR (RE-Approach) ☒ Monte Carlo

Evaluation Parameters

Value Date 10.03.2007 ☐ Incremental risk ☒ Forex
Eval. Curr. EUR ☐ Marginal risk ☒ Share
Output Curr. EUR ☒ Interest
☒ Rate
☒ Total

Default to Grid Corr. 0,00 Default to Default Corr. 0,00

Monte Carlo Parameters

Number of runs 4000 Intervals 100
Horizon days 1 Auto seed ☐ Seed 0

PMS Volatility Data Scenario

Method Basel - Plain StDev. Dataset Date 09.03.2007

Conf. Interval 99,00 % Multiplier for Standard Deviation 2,33

VaR 202,17

Additional Features

☐ Delta Plus

MC-Results

	Histogram	Series	Mean	949.717,27
Standard Deviation	88,10		Min Value	949.426,10
Skewness	0,04		Max Value	950.032,35
Kurtosis	-0,01		Range	606,25

Total risk Build time: 4922 Calc time: 32797

Fig. 7 Market Risk Evaluation Dialog

Ser Num	Action	Include (Mark)	Not Amount	Coupon	Instrument Type	Curr	Total VaR	Incremental VaR	Marginal VaR
209896	Long	*	1 000 000,00	4,00%	Bond	EUR	7 056,10	7 056,10	-70,56
A0AHHM	Long		1 000 000,00	5,50%	Bond	EUR			
Portfolio							7 056,10		
209896	Long	*	1 000 000,00	4,00%	Bond	EUR			
A0AHHM	Long	*	1 000 000,00	5,50%	Bond	EUR	12 584,33	12 584,33	-125,84
Portfolio							12 584,33		
209896	Long	*	1 000 000,00	4,00%	Bond	EUR	6 929,04	5 860,17	-79,16
A0AHHM	Long	*	1 000 000,00	5,50%	Bond	EUR	12 647,90	11 515,89	-105,28
Portfolio							18 444,50		

Fig. 8 Excel Export of Total Margin and Incremental VaR

3.4 Simulation Parameters

The following main market simulation parameters are used to control the Monte Carlo simulation (s. Fig. 7):

- Number of Monte Carlo runs: usually 4.000, but a larger number of runs is possible.
- Horizon days: holding period, which is 1 day by default, can be set to 10 days, for example.
- Valuation Date and Date of Correlation Matrix in the data base.
- Confidence Level (for example, 99%) and corresponding multiplier for standard deviation

3.5 Price Distributions and Drilldowns

The Monte Carlo simulation generates price distributions (s. Fig. 9) and undertakes measures for every hierarchical level, according to the risk types above:

- On position level: single Position VaR, VaR-ES, FX-VaR, Marginal and Incremental VaR, etc.
- Measures on position level, for portfolio and subportfolio level, aggregate according to the portfolio structure in PMS. In the course of the simulation run, the calculation is performed on all hierarchical levels simultaneously, because of pricing expressions that represent position prices, but are extended by the sum to represent the entire portfolio structure. During simulation runs, specific portfolio pricing nodes are marked, to accumulate results for distributions .

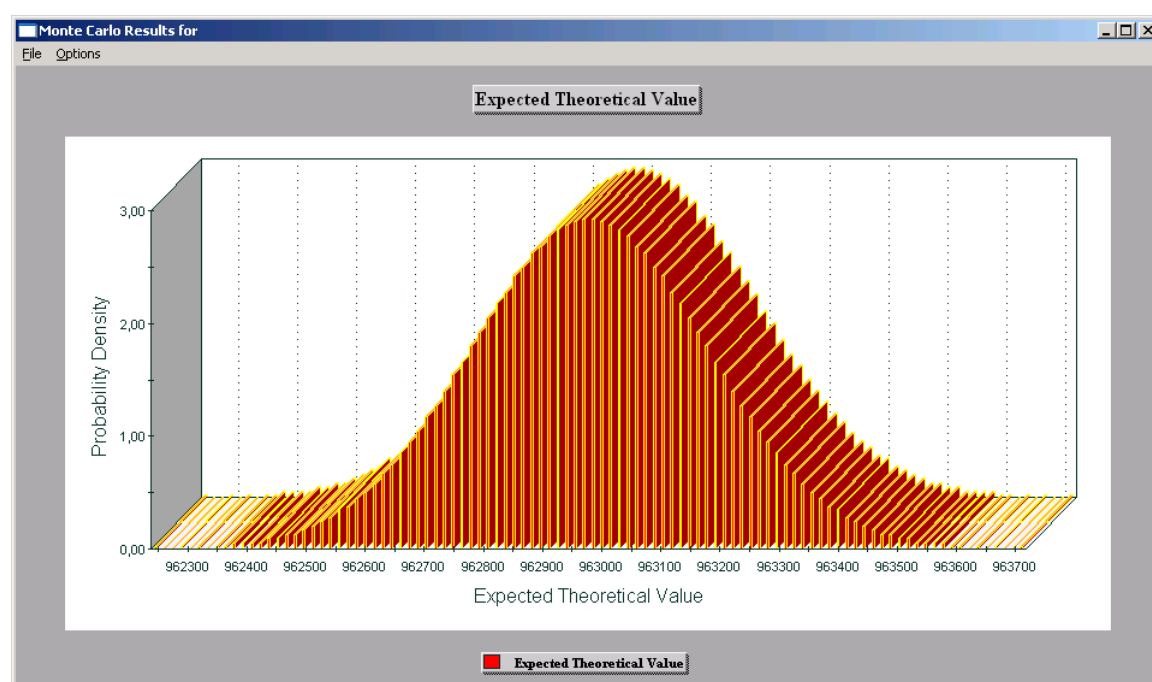


Fig. 9 Price Distribution of Portfolio

For all data, generated before or during the Monte Carlo Simulation, each risk driver can be displayed in a separate window (s. Fig. 10). Correlated random or result series can be exported to Excel for further analysis.

It is possible to select a „worst case“ (red marked value row), i.e. left from confidence value, to treat it as a stress test scenario. All correlated changes regarding risk drivers can be stored as a scenario in the PMS scenario manager section for subsequent usage.

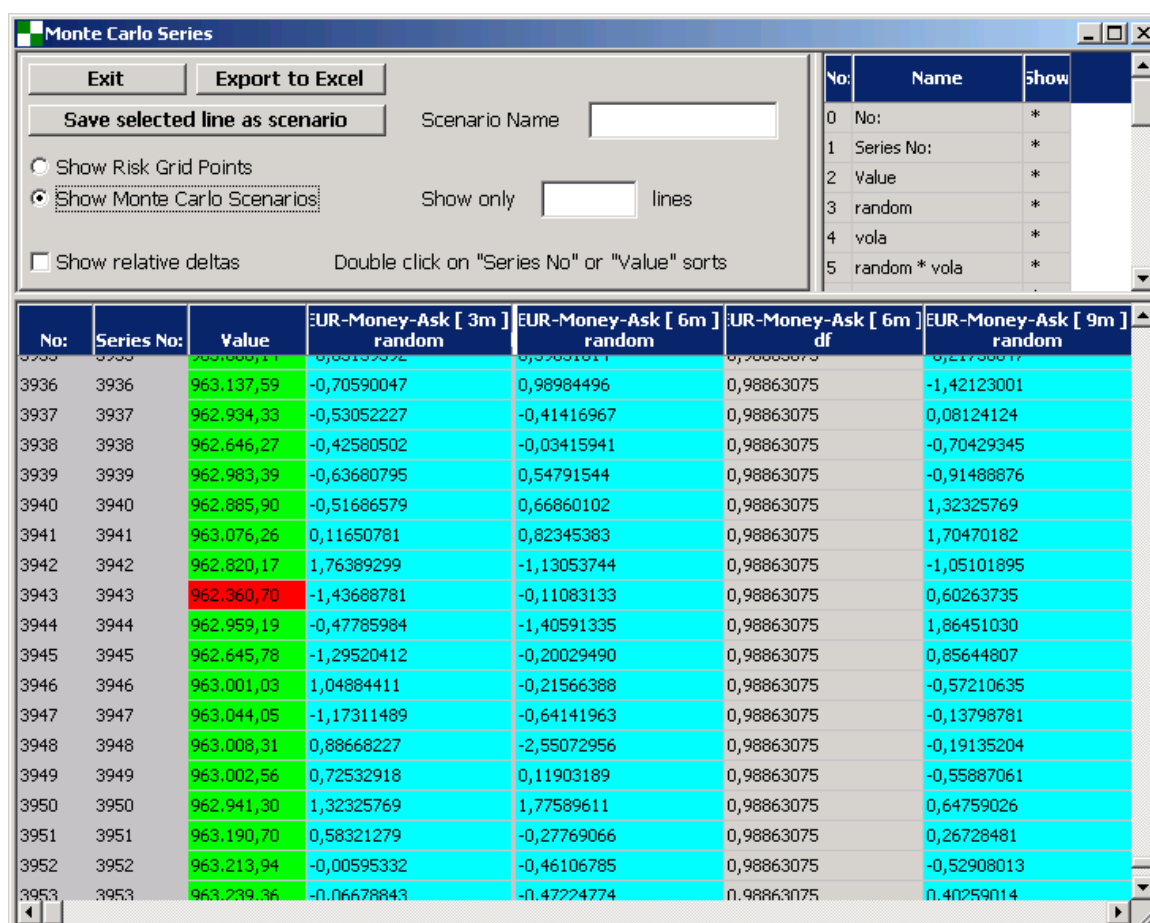


Fig. 10 Chart of Monte Carlo Series and Prices

4. Detailed Example of VaR Calculations using Monte Carlo Simulations

4.1 General Remarks on Monte Carlo simulations

The calculation of market VaR for financial instruments using Monte Carlo simulation is based on a set of approaches and assumptions:

- A position's market VaR in a financial instrument is a probable negative move of the position's price value, resulting from a negative correlated move of underlying market factors, such as interest rates, foreign exchange rates or spreads, which influence the price value.
- The negative move of the price value is accounted for a given risk horizon (for example, 1 or 20 days from the valuation date) and for a selected confidence level (for example 95% or 99%).
- The position's price value at valuation date is known, but for the risk horizon, the price value can be estimated by a distribution of the price value, having an expected value and a confidence value (i.e. a value, left from distribution, corresponding to confidence level). The VaR of the position (or portfolio) is then given by:

$$\text{VaR} = \text{expected value} - \text{confidence value}$$

- Monte Carlo simulation generates the distribution of price values on the risk horizon, calculating different price values for a large set (for example 5000) of correlated scenarios of underlying market factors. These 5000 price values represent the needed price distribution.
- The first step in running a Monte Carlo simulation on a position (or portfolio) is the representation of a pricing model (formulas to calculate the price value) in an effective way, so that this model can be calculated 5000 times with different market conditions.
- An effective representation of the pricing formulas lies within an arithmetic expression tree, that includes arithmetic operators, such as -, +, /, *, stochastic variables for the market factors, as well as constants and option elements to represent option rights.
- The following step includes the generation of a correlation matrix and volatility vector for all required market factors from the historic series. The matrix and vector are then used to create large sets of synthetic scenarios for the market factors.

- The scenarios for market factors are then used to simulate the pricing model, where, for each Monte Carlo run, a set of small correlated changes (scenarios) is applied on market factors within the tree representation.
- The results, generated from the pricing model, are stored for every Monte Carlo run and are used to represent the price distribution, as well as a basis for the calculation of expected value and VaR.

The following section provides a detailed description of the presentation and simulation of an exemplar Swaption instrument, demonstrating swaption conditions, pricing model, pricing expression tree, simulation and VaR results.

4.2 Example Instrument: Swaption

4.2.1 Contractual Agreement for Swaptions

A Swaption is a swap option that grants its owner the right, but not the obligation, to enter into an underlying swap on the option's exercise date. The term "Swaption" typically refers to interest rate swap options. There are two types of swaption contracts:

- A payer swaption gives the owner of the swaption the right to enter into a swap where they pay the fixed leg and receive the floating leg.
- A receiver swaption gives the owner of the swaption the right to enter into a swap where they will receive the fixed leg, and pay the floating leg.

The buyer and seller of the swaption agree on:

- the premium (price) of the swaption
- the strike rate (equal to the fixed rate of the underlying swap)
- length of the option period (which usually ends two business days prior to the start date of the underlying swap),
- the terms of the underlying swap,
- the currency of Swaption and underlying swap
- notional amount,
- amortization, if any
- frequency of payment settlements of the underlying swap

Swaptions can be viewed as options on the "fixed" side of a swap. This follows the logic of the theoretical assumption of a liquid market, in which it is possible to enter into a swap with a fixed coupon, that is equal to the forward swap rate, at any time, without additional costs.

4.2.2 Pricing Approach for Swaptions

Swaptions can be priced using the Black-model, that is based on the following assumptions:

- In a liquid market, it is possible – at any time T – to enter the "fair" payer or receiver swap at the forward swap rate. By definition, F is the (Forward) swap rate, recruiting, from today's perspective, the underlying swap as fair (swap price is zero). In other words, forward values of fixed and variable swap sides are equal.
- This results in the payoff function of a receiver swaption – which depends on forward swap rate F and strike rate S (equal to the fixed rate of the underlying swap) – looking as follows:

$$\text{PayoffReceiver } (F) = \text{cash-flow payment of the fixed swap side is the max} \\ (S - F, 0)$$

Payments from the variable side, from the receiver swaption and the corresponding income from the payer swap, cancel each other out. What is remaining are the negative payments from the Payer Swap ($-F$) and the revenues from the receiver swap (S). Since this structure is optional, the option holder will choose this construct only if $S > F$; otherwise, he won't exercise the option and contract the underlying swap.

With the usual assumptions of the Black-model (the Volatility of F is log-normally distributed with σ), the price of the swaption can be calculated as the expected value of the payoff function, discounted to the valuation date:

$$\text{PreisRecSwaption } (F) = \text{Nominal} * \text{PriceFactor} * (S * N(-d_2) - F * N(-d_1)),$$

where:

Nominal	nominal value of the swaptation (and the underlying swap)
Price Factor	$\text{Sum } (DF(t_i)) * \tau / \text{Number of Payments per Year}$
$DF(t_i)$	discount factor for cash flow date of $\text{Cashflow}_{(i)}$
τ	period length of the fixed side of the swap
$N(.)$	standard normal distribution function
d_1	$\ln((F / S) + \sigma^2 T / 2) / (\sigma \sqrt{T})$, and
d_2	$d_1 - \sigma \sqrt{T}$
T	duration of the swaption
S	strike rate of the swaption (coupon of the swap's fixed side)

F forward rate of the swap
 o volatility of F p.a.

The price for a payer swaption is given by:

$$PreisPaySwaption(F) = Nominal * PriceFact_{or} * (F N(d_1) - S N(d_2))$$

Calculation of the swap's forward rate F is based on the fair condition:

$PV(Float Side) = PV(Fix Side)$, i.e.

$$DF(t_{Start}) - DF(t_{End}) = \sum_{i=Start+1}^{i=End} F * dt(t_{i-1}, t_i) * DF(t_i) \quad F = \frac{(DF(t_{Start}) - DF(t_{End}))}{\sum_{i=Start+1}^{i=End} dt(t_{i-1}, t_i) * DF(t_i)}$$

where:

DF(t_i) discount factor for time point i
 t_i end date of period i
 dt(t_{i-1}, t_i) period length between i-1 and i
 t_{Start}, t_{End} start and end date of the swap

The above pricing model includes the following elements into the pricing formulas:

1. Stochastic elements:
 Discount Factor nodes DF(i) for the Price Factor and for the forward rate F Black-Model node using S, F and σ (for the duration T it is $\sigma \sqrt{T}$)
1. Non-stochastic elements:
 Constant nodes (period length, Nominal, strike S, duration T in years) Arithmetic operators nodes (+, -, /, *)

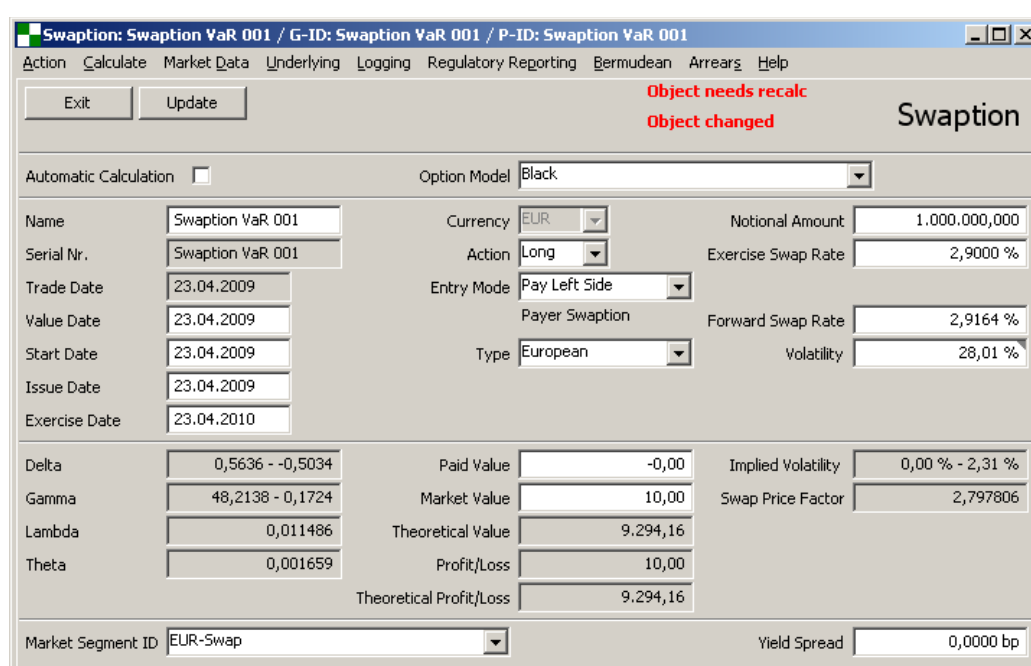
4.2.3 Data of an example Swaption

Figure 11 shows data and static results of an example swaption explaining the simulation approach. The following contract data are important in terms of the pricing model:

- Nominal Amount = 1 000 000 in EUR
- Exercise Swap Rate S = 2.9%, Forward Swap Rate F = 2.9164%
- Market Volatility σ of the Forward Swap Rate F is 28.01%
- Position is Long, i.e. we should pay the Option Value

- The underlying swap is a payer left side swap (the fix side) = payer swaption
- Market factors are zero discount factors of the evaluation curve EUR-Swap.
- The life of the Swaption lasts exactly 1 year; the life of the swap last exactly 3 years; so relevant market factors are discount factors on the points of 1, 2, 3 and 4 years

Contractual conditions of the swaption and the underlying swap are simplified (equal frequencies, 1 year periods, Day count = 30/360) for a better understanding of the simulation approach. Figures 12, 13 and 14 show the data of the underlying swap.



Swaption: Swaption VaR 001 / G-ID: Swaption VaR 001 / P-ID: Swaption VaR 001

Action Calculate Market Data Underlying Logging Regulatory Reporting Bermudean Arrears Help

Exit Update Object needs recalc
Object changed **Swaption**

Automatic Calculation ☐ Option Model Black

Name	Swaption VaR 001	Currency	EUR	Notional Amount	1.000.000,000
Serial Nr.	Swaption VaR 001	Action	Long	Exercise Swap Rate	2,9000 %
Trade Date	23.04.2009	Entry Mode	Pay Left Side		
Value Date	23.04.2009	Payer Swaption		Forward Swap Rate	2,9164 %
Start Date	23.04.2009	Type	European	Volatility	28,01 %
Issue Date	23.04.2009				
Exercise Date	23.04.2010				

Delta	0,5636 - 0,5034	Paid Value	-0,00	Implied Volatility	0,00 % - 2,31 %
Gamma	48,2138 - 0,1724	Market Value	10,00	Swap Price Factor	2,797806
Lambda	0,011486	Theoretical Value	9.294,16		
Theta	0,001659	Profit/Loss	10,00		
		Theoretical Profit/Loss	9.294,16		

Market Segment ID: EUR-Swap Yield Spread: 0,0000 bp

Fig. 11 Definition of example Swaption

Swap: Und. of Swaption <<No Name>> / G-ID: Swaption VaR 001@u as Underlying

Action Calculate Edit Market Data Logging Regulatory Reporting Arrears Help Origin

Exit Update Left Side Right Side Swap

Payer-Swap

Name	Und. of Swaption <<No Narr	Pay/Rec	Pay Left Side	BP Value	-281,24
Value Currency	EUR	Amortization	No Amortization	Duration	175,7905
Evaluation Date	23.04.2009	Market Clean-Value	100,00	Proceeds	0,00
Start Date	23.04.2010	NPV	459,61	Accr. Interest	0,00
Effective Date	23.04.2010	Th. Clean-Value	459,61		
Maturity	23.04.2013				

Pay Date	Total CF	Net Present Value
23.04.2011	-6.647,30	-6.395,29
23.04.2012	1.211,34	1.131,15
23.04.2013	6.346,19	5.723,75

Fig. 12 Underlying Swap

Left Side of Swap: Und. of Swaption <<No Name>> / G-ID: Swaption VaR 001@u as Underlying

Action Calculate Edit Conventions Market Data Regulatory Reporting Help Origin

Swap Other Side Adjust NA Calc Fair Rate clean Left Side

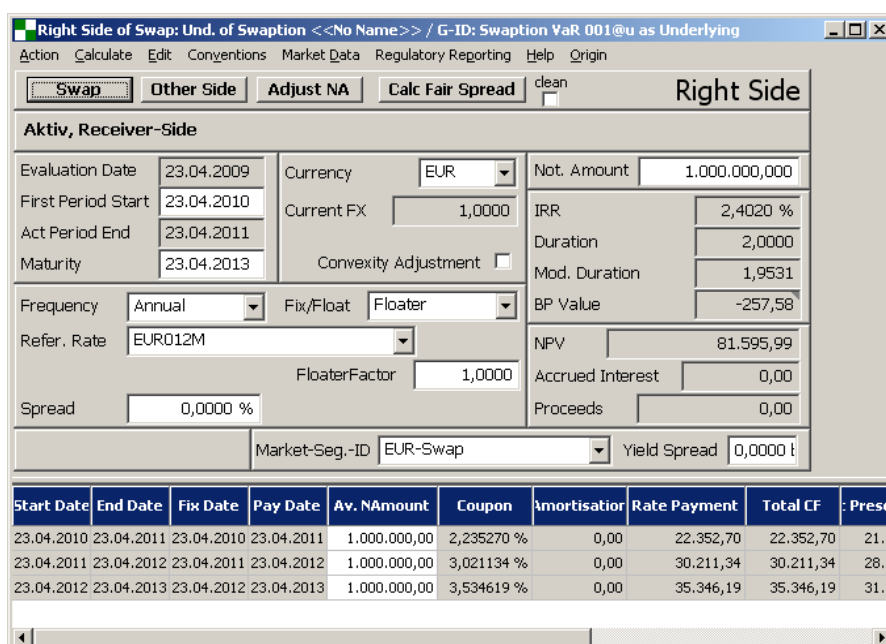
Passiv, Payer-Side

Evaluation Date	23.04.2009	Currency	EUR	Not. Amount	1.000.000,00
First Period Start	23.04.2010	Current FX	1,0000	IRR	2,3593 %
Act Period End	23.04.2011			Duration	2,9845
Maturity	23.04.2013			Mod. Duration	2,9157
Frequency	Annual	Fix/Float	Fix	BP Value	-23,66
Interest Rate	2,9000 %			NPV	-81.136,38
				Accrued Interest	-0,00
				Proceeds	-0,00

Market-Seg.-ID EUR-Swap Yield Spread 0,0000 %

Start Date	End Date	Pay Date	Av. NAmount	Coupon	Amortisation	Rate Payment	Total CF	Net Present Value	Pro
23.04.2010	23.04.2011	23.04.2011	1.000.000,00	2,900000 %	0,00	29.000,00	-29.000,00	-27.900,56	
23.04.2011	23.04.2012	23.04.2012	1.000.000,00	2,900000 %	0,00	29.000,00	-29.000,00	-27.080,16	
23.04.2012	23.04.2013	23.04.2013	1.000.000,00	2,900000 %	0,00	29.000,00	-29.000,00	-26.155,66	

Fig. 13 Left fixed side of the swap



Right Side of Swap: Und. of Swaption <<No Name>> / G-ID: Swaption VaR 001@u as Underlying

Buttons: Swap, Other Side, Adjust NA, Calc Fair Spread, clean

Right Side

Aktiv, Receiver-Side

Evaluation Date	23.04.2009	Currency	EUR	Not. Amount	1.000.000,000
First Period Start	23.04.2010	Current FX	1,0000	IRR	2,4020 %
Act Period End	23.04.2011	Convexity Adjustment	<input type="checkbox"/>	Duration	2,0000
Maturity	23.04.2013	Frequency	Annual	Mod. Duration	1,9531
		Fix/Float	Floater	BP Value	-257,58
		Refer. Rate	EUR012M	NPV	81.595,99
		FloaterFactor	1,0000	Accrued Interest	0,00
		Spread	0,0000 %	Proceeds	0,00
		Market-Seg.-ID	EUR-Swap	Yield Spread	0,0000 %

Start Date	End Date	Fix Date	Pay Date	Av. NAmount	Coupon	Amortisation	Rate Payment	Total CF	Prese
23.04.2010	23.04.2011	23.04.2010	23.04.2011	1.000.000,00	2,235270 %	0,00	22.352,70	22.352,70	21.5
23.04.2011	23.04.2012	23.04.2011	23.04.2012	1.000.000,00	3,021134 %	0,00	30.211,34	30.211,34	28.2
23.04.2012	23.04.2013	23.04.2012	23.04.2013	1.000.000,00	3,534619 %	0,00	35.346,19	35.346,19	31.8

Fig. 14 Right floater side of the swap

4.2.4 Representation of the Swaption example on a Pricing Tree

The pricing model, explained above, can be rewritten for the swap example as follows:

$$\text{PaySwaptionValue}(\text{EUR}) = \text{Nominal}(\text{EUR}) * \text{PiceFactor} * \text{BlackCallOption}(F, S, \sigma(T))$$

$$\text{PiceFactor} = \text{DF}(\text{EUR-Swap}, 2Y) + \text{DF}(\text{EUR-Swap}, 3Y) + \text{DF}(\text{EUR-Swap}, 4Y)$$

$$F = (\text{DF}(\text{EUR-Swap}, 1Y) - \text{DF}(\text{EUR-Swap}, 4Y)) / \text{PiceFactor}$$

where:

- $\tau = 1$ period length of the fixed side of the swap
- $n = 3$ number of swap periods (from years 1 to 4)
- $T = 1$ duration of the swaption
- $\sigma(T) = \sigma \sqrt{T}$ volatility of the forward swap rate

After replacing the swaption data from the example with the formulas above, formulas are derived that can be represented by a fast pricing tree used in Monte Carlo simulations:

$$\text{PaySwaptionValue}(\text{EUR}) = 1\,000\,000 * \text{PiceFactor} * \text{BlackCallOption}(F, 0.029, 0.2801)$$

$$\text{PiceFactor} = \text{DF}(\text{EUR-Swap}, 2Y) + \text{DF}(\text{EUR-Swap}, 3Y) + \text{DF}(\text{EUR-Swap}, 4Y)$$

$$F = (\text{DF}(\text{EUR-Swap}, 1Y) - \text{DF}(\text{EUR-Swap}, 4Y)) / \text{PiceFactor}$$

The corresponding pricing tree is shown in Fig. 15.

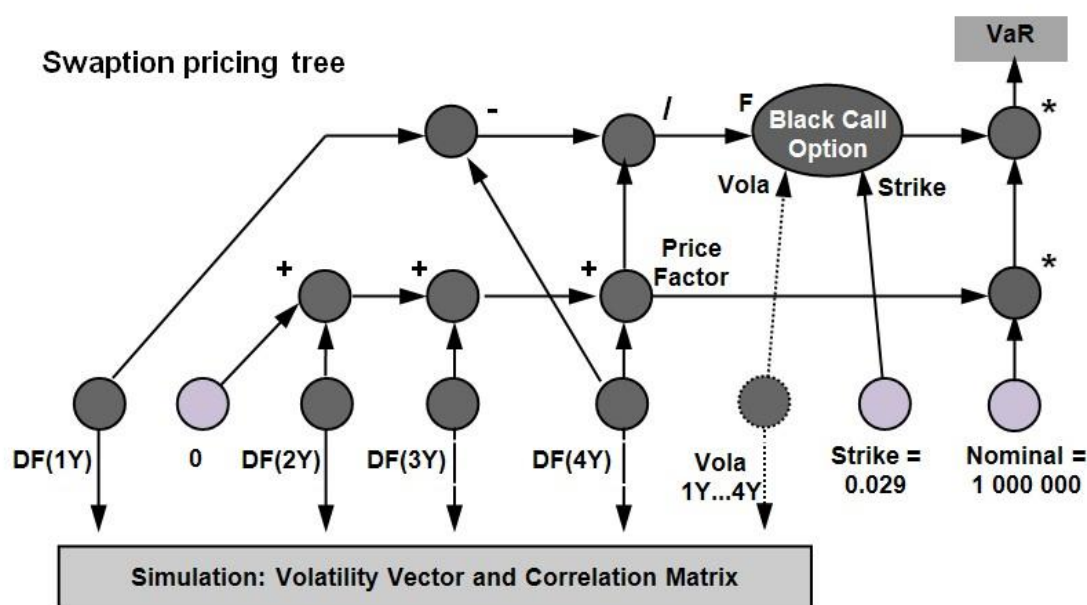


Fig. 15 Pricing Tree Example

The Price Factor is the sum of discount factor nodes; the Forward Swap Rate F makes use of the Price Factor as divisor. The Black Call Option is applied to the Forward Swap Rate F against the Strike and assumes the volatility of the Forward Swap Rate F . The Option Price is then multiplied by the Nominal and the Price Factor. Except for the Option node, the tree representation is based only on two-operand arithmetic operators, which are elements of the pricing formula.

4.3 Simulation of the Swaption

The Simulation is based on the Monte Carlo simulation of scenarios, for a large set of runs, for example 4000. Monte Carlo scenarios are obtained from the Correlation Matrix and Volatility Vector for market factors, that are gained from historical statistics over their time series, as shown in Section 2. The correlation matrix and volatility vector for the swaption example are given below. The volatility was calculated on a daily basis. The VaR simulation in the example was performed without including the Vola risk factor, assuming zero volatility of the implied volatility. For this reason, the Vega part of the swaption risk will be ignored. In case the Vega risk is included, there should be a historical time series for the implied volatility and a corresponding volatility and correlation included into the missing rows and columns of the tables below.

Correlation Matrix for Swaption Simulation

	EUR-Swap 1 Year	EUR-Swap 2 Years	EUR-Swap 3 Years	EUR-Swap 4 Years	Implied Volatility
EUR-Swap 1 Year	1	0.2215	0.2564	0.1266	...
EUR-Swap 2 Years	0.2215	1	0.8202	0.7799	...
EUR-Swap 3 Years	0.2564	0.8202	1	0.9135	...
EUR-Swap 4 Years	0.1266	0.7799	0.9135	1	...
Implied Volatility	1

Volatility Vector for Swaption Simulation

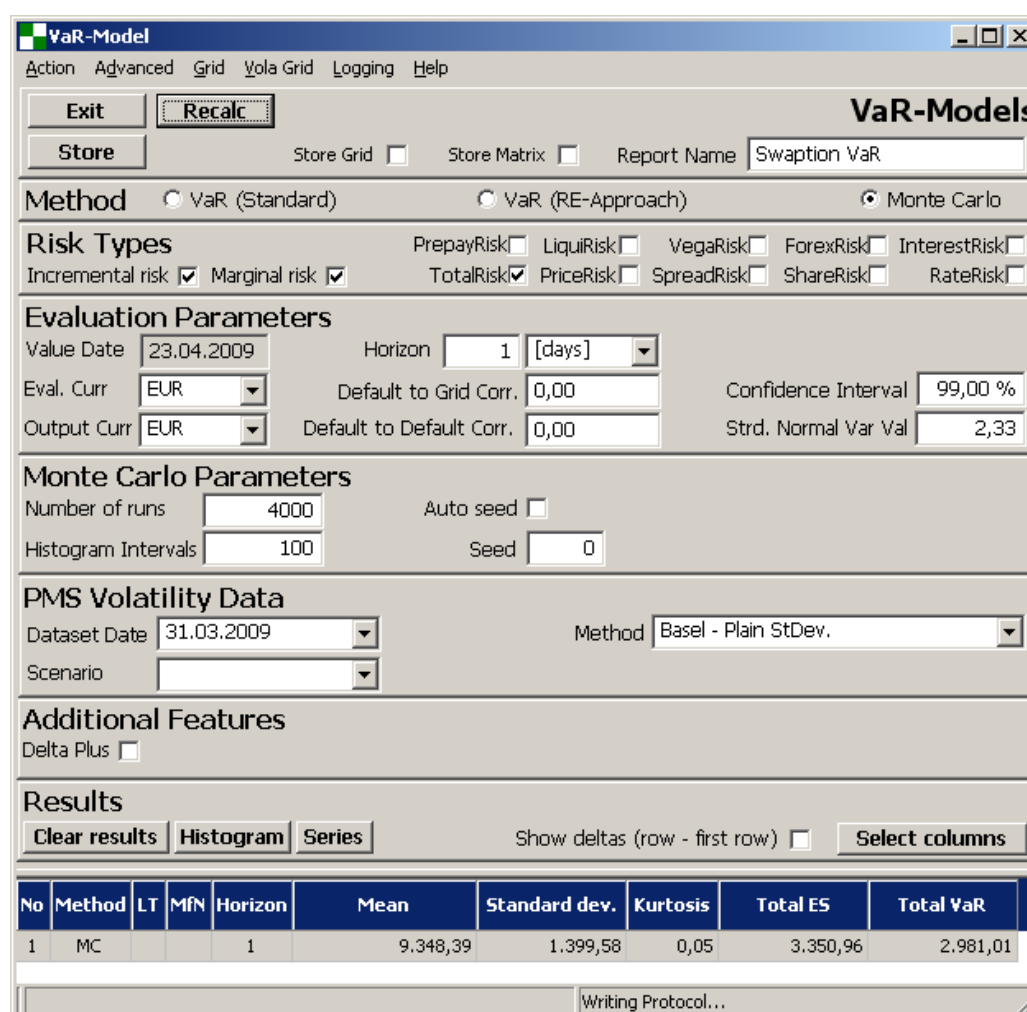
Market Factor	Matrix Risk Grid Point	Volatility
EUR-Swap 1 Year	Money 42	0.00029899
EUR-Swap 2 Years	Swap 182	0.00132174
EUR-Swap 3 Years	Swap 183	0.00198575
EUR-Swap 4 Years	Swap 184	0.00264766
Implied Volatility

The simulation is performed using the following steps:

- Set up the simulation parameters on Fig. 16. (in the example simulation to 4000 runs, Confidence level = 99%, Horizon of risk = 1 day). The simulation process is started by pressing the Recalc button.
- The correlation and volatility for required risk factors of the swaption are extracted from a large correlation matrix on 31.03.2008. The correlation matrix and volatility vector are calculated in advance and are based on the historical time series.
- Market factors are not market rates; the corresponding discount factors are used as market factors, which simplifies the simulation. Discount factors are calculated from market rates, using curve bootstrapping prior to running the statistics and calculation of the correlation and volatility of the discount factor's daily performance.
- Before the simulation, a set of 4000 normally distributed and correlated scenarios is calculated for all risk factors. This is, in fact, a matrix of 4000 x 4 of normally distributed

and correlated random samples with a mean of 0 and a volatility of 1. A vast number of corrections of random samples is performed (s. 2.2.5 above).

- The pricing tree for the swaption, according to Fig. 15., is constructed in the memory. This is, in fact, a representation of the pricing formula for the swaption that will be simulated, i.e. the pricing formula is calculated 4000 times. The tree representation allows for very fast calculation, because the pricing routine that is being interpreted is small and can be held in the processor cache memory.



VaR-Model

Action Advanced Grid Yola Grid Logging Help

Exit Recalc Store

Store Grid ☐ Store Matrix ☐ Report Name Swaption VaR

Method ☐ VaR (Standard) ☐ VaR (RE-Approach) ☒ Monte Carlo

Risk Types

Incremental risk ☒ Marginal risk ☒ PrepayRisk ☐ LiquiRisk ☐ VegaRisk ☐ ForexRisk ☐ InterestRisk ☐ TotalRisk ☒ PriceRisk ☐ SpreadRisk ☐ ShareRisk ☐ RateRisk ☐

Evaluation Parameters

Value Date 23.04.2009 Horizon 1 [days]

Eval. Curr EUR Default to Grid Corr. 0,00 Confidence Interval 99,00 %

Output Curr EUR Default to Default Corr. 0,00 Strd. Normal Var Val 2,33

Monte Carlo Parameters

Number of runs 4000 Auto seed ☐ Histogram Intervals 100 Seed 0

PMS Volatility Data

Dataset Date 31.03.2009 Method Basel - Plain StDev.

Scenario

Additional Features

Delta Plus ☐

Results

Clear results Histogram Series Show deltas (row - first row) ☐ Select columns

No	Method	LT	MfN	Horizon	Mean	Standard dev.	Kurtosis	Total ES	Total VaR
1	MC			1	9.348,39	1.399,58	0,05	3.350,96	2.981,01

Writing Protocol...

Fig. 16 Monte Carlo Simulation of the example Swaption

- Prior to the simulation, the actual static values of market factors are applied to market factors in the tree (in the example, these are the discount factors for 1, 2, 3 and 4 years, and, possibly, the implied volatility).

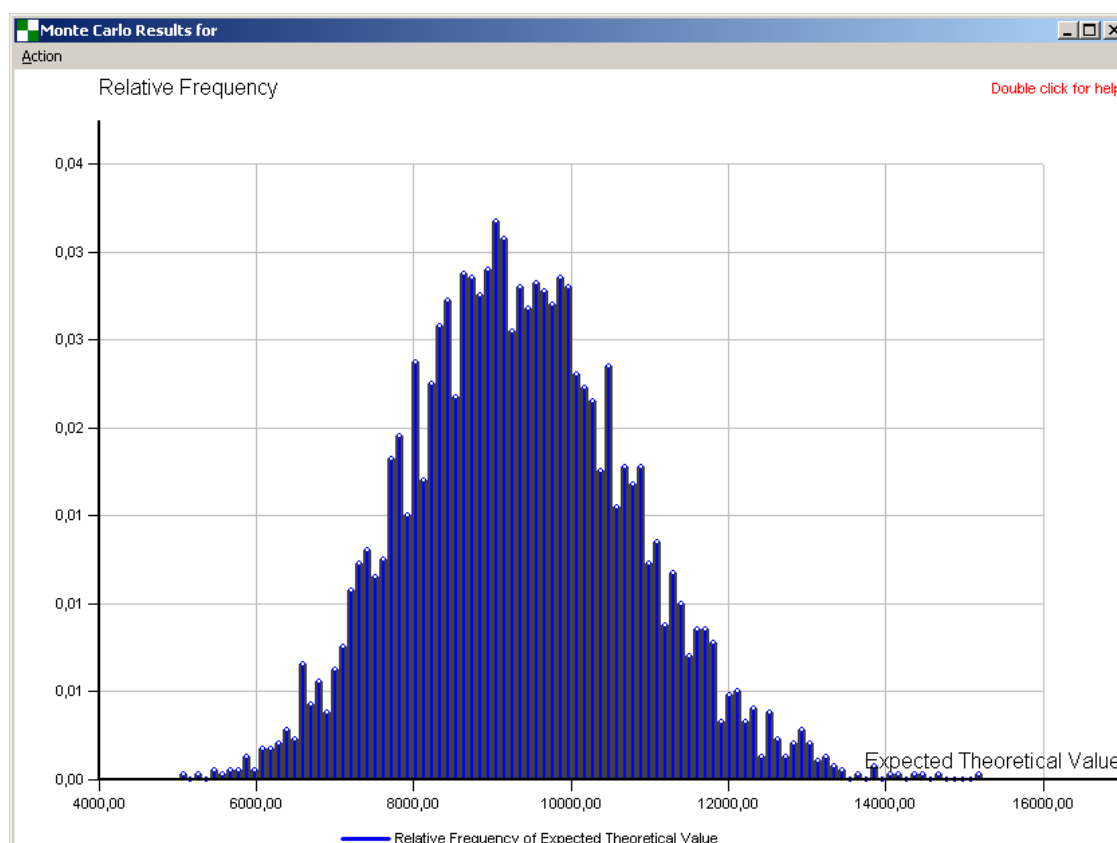
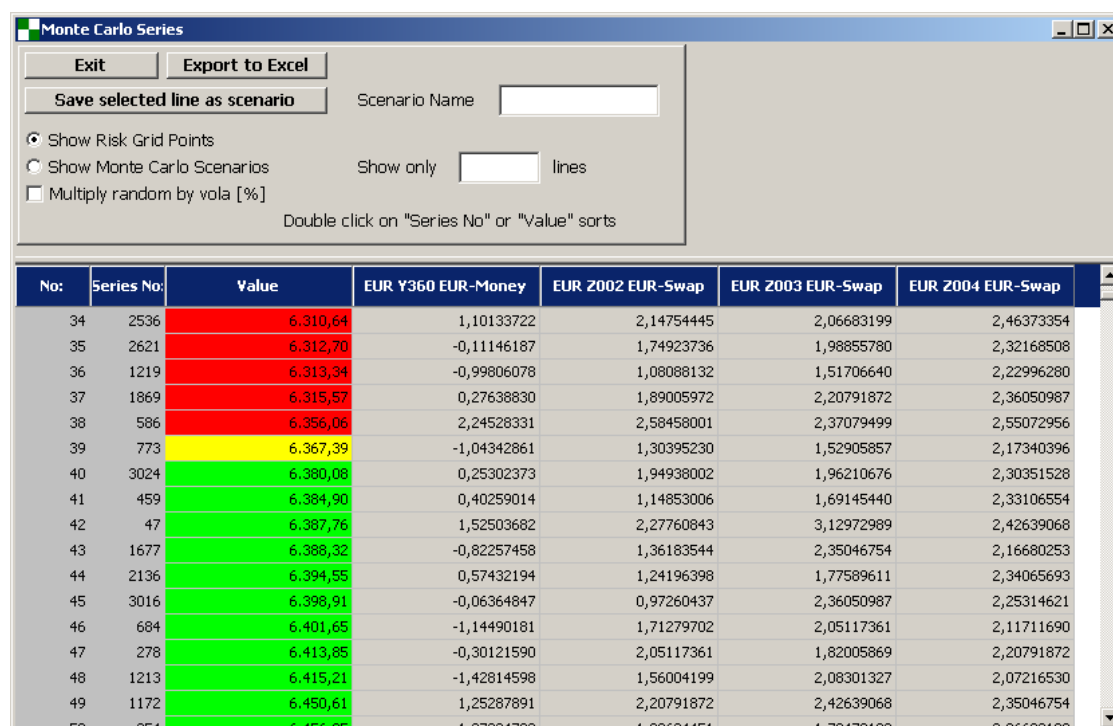


Fig. 17 Simulation Distribution of the Swaption price series

- For every Monte Carlo run, the module obtains the next set (row) of normal distributed and correlated samples, multiplies this vector with the volatility vector and applies the small changes exponentially as scenarios on the actual static values. The entire tree is then calculated in a recursive procedure, obtaining the price on the top node of the tree. This price is stored, so that, after 4000 runs, the price series of swaption prices can be used to give the resulting distribution and statistics.
- The distribution mean and VaR are obtained from the swaption price series (s. Fig. 17.) The distribution is skewed and not normal; it should be close to log-normal, because of the used Black-model.
- The distribution mean (the expected value) is 9348.39 and is close to a static calculation of 9294,16 (Theoretical Value on Fig. 11.). VaR is calculated from the distribution to 2982.01. The calculation of the confidence value is simple, 1% (100% - 99%) of 4000 prices is 40 prices. The price series is sorted in ascending order, so the 40-en price is the confidence value and VaR can thereupon be interpreted as: confidence value – expected value.
- The price values, according to the scenarios, are represented in Fig. 18. For every set of correlated samples of the 4 factors, a new price value is calculated. The

confidence value (the yellow row in Fig. 18.) is obtained after ordering the price value series.

- Fig. 19. shows a detailed representation of primary risk factor series (market interest rates), as well as secondary risk factors series (discount factors obtained after bootstrapping each historical market curve within the historical period, usually 1 year).



The screenshot shows the 'Monte Carlo Series' window. It includes buttons for 'Exit', 'Export to Excel', and 'Save selected line as scenario'. There are also checkboxes for 'Show Risk Grid Points', 'Show Monte Carlo Scenarios', and 'Multiply random by vola [%]'. A 'Show only' field is set to 'lines'. Below the controls is a table with the following data:

No:	Series No:	Value	EUR Y360 EUR-Money	EUR 2002 EUR-Swap	EUR 2003 EUR-Swap	EUR 2004 EUR-Swap
34	2536	6.310,64	1,10133722	2,14754445	2,06683199	2,46373354
35	2621	6.312,70	-0,11146187	1,74923736	1,98855780	2,32168508
36	1219	6.313,34	-0,99806078	1,08088132	1,51706640	2,22996280
37	1869	6.315,57	0,27638830	1,89005972	2,20791872	2,36050987
38	586	6.356,06	2,24528331	2,58458001	2,37079499	2,55072956
39	773	6.367,39	-1,04342861	1,30395230	1,52905857	2,17340396
40	3024	6.380,08	0,25302373	1,94938002	1,96210676	2,30351528
41	459	6.384,90	0,40259014	1,14853006	1,69145440	2,33106554
42	47	6.387,76	1,52503682	2,27760843	3,12972989	2,42639068
43	1677	6.388,32	-0,82257458	1,36183544	2,35046754	2,16680253
44	2136	6.394,55	0,57432194	1,24196398	1,77589611	2,34065693
45	3016	6.398,91	-0,06364847	0,97260437	2,36050987	2,25314621
46	684	6.401,65	-1,14490181	1,71279702	2,05117361	2,11711690
47	278	6.413,85	-0,30121590	2,05117361	1,82005869	2,20791872
48	1213	6.415,21	-1,42814598	1,56004199	2,08301327	2,07216530
49	1172	6.450,61	1,25287891	2,20791872	2,42639068	2,35046754
50	254	6.456,86	1,27324703	1,00634451	1,70470183	2,06683199

Fig. 18 Simulation of risk factors and calculation of the price value series

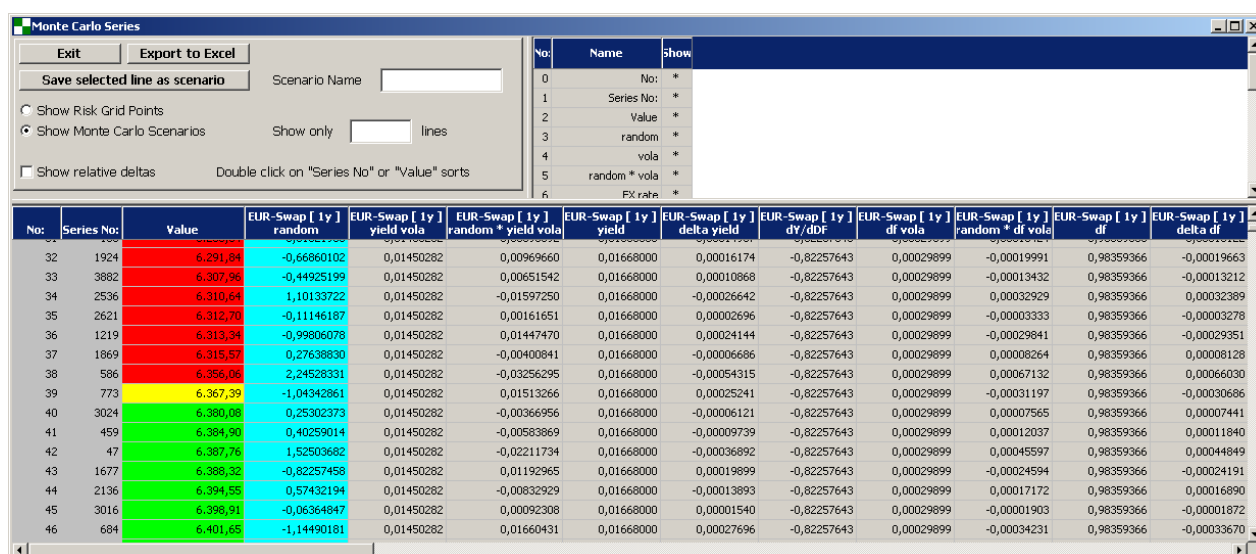


Fig. 19 Detailed representation of the risk factors

5. Spread VaR Calculation using Monte Carlo Simulation

5.1 Further Remarks to the Monte Carlo Simulation

The calculation of market VaR for financial instruments using Monte Carlo Simulation is based on a set of approaches and assumptions given in section 4.1 General Remarks to Monte Carlo Simulation. Further remarks are outlined below:

The pricing formula, that is implemented as a closed formula in the simulation tree, can be different from the one in the pricing model of static price calculation. The reasons for this may be multiple:

- Only a small set of RGP (Risk Grid Points) represent the interest rate risks, i.e. 1 Mo, 3 Mo, .., 1Y, 2Y, .. . For these RGPs there are certain volatilities and correlations in the market risk matrix. Thus,, all intermediate cash flows are mapped on the existing grid points, splitting on the left and the right GRP. This splitting is neutral related to the volatility and the PV of the original cash flow, but at skewed Monte Carlo distributions, the expected value does not exactly achieve the static PV.
- Only a small set of RGP (Risk Grid Points) represents the credit spread risks. Splitting, similar to the one for interest rates, is used for intermediate future time points, where cumulated default probabilities, i.e. PDs along the time axis, are needed.
- Options are represented by Black-Nodes and no other option types, (exotic, path dependent, etc.) can be simulated. These options are mapped to Black-Options.
- Instruments that use a numeric pricing model (such as HW) cannot be simulated on the simulation tree, because of the performance and the missing closed pricing formulas. These instruments are approximated by key rate sensitivities and a restricted Taylor expansion.

This section describes the presentation and simulation of an example asset instrument, which embeds credit risk involved by the CDS spread curve connected to the instrument. The pricing model for the expected loss, the pricing expression tree, the simulation and the VaR results are represented.

5.2 Pricing Approach for Spread Risk

The spread risk calculation for assets is based on the calculation of expected loss and uses the following approaches:

- Spread risk is a credit risk, measured with the premium changes for credit default swaps on the CDS market.
- It is a risk of deterioration in the creditworthiness of the Reference Entity, while the CDS increases premiums and consequently the hedging costs. Accordingly, the risk of losses will also increase, so the spread risk as unexpected loss will be equal to the risk of expected loss.
- Fair Value (asset) = risk free value (asset) – Expected Loss (asset),

where

Risk free value (asset) is calculated with the risk free curve, and
Expected Loss (asset) is calculated with the spread curve

- On the CDS market, the fair price of CDS = 0 fixes the fair premium for each maturity.
- Reference entities are countries, firms, concerns, CDS indices (CDX: iTraxx ® Europe).
- Spread is the premium of a credit default swap. The Spread Curve incorporates premiums for standardized maturities: 6Mo, 1, 2, 3, 4, 5, 7, 10 years.
- The expected loss calculation is performed using a periodic approach:

Expected Loss = $\sum(\text{Expected Loss (i)} * \text{Discount Factor(i)})$

Expected Loss (i) = $\text{EAD(i)} * \text{PD(i)} * \text{LGD(i)}$

Expected Loss (i) = $\text{NA} * \text{Marginal PD(i)} * (1 - \text{Recovery Rate})$

Marginal PD(i) = $\text{Cumulative PD(i)} - \text{Cumulative PD(i-1)}$

where:

EAD(i)	Exposure of the asset at default for Period i
PD(i)	Probability of default for Period i
LGD(i)	Loss given default for Period i
Cumulative PD(i)	PD(0,i) Probability of default from T0 to end of Period i
Discount factor(i)	DF(0,i) Risk free discount factor to end of Period i

- According to the above stated formulas, the risk of the expected loss (spread risk) depends on changes in Cumulative PDs, as well as changes in Discount Factors. So, the spread risk depends on risk free evaluation curves and asset specific spread curves or spread indices.
- A spread index is interpreted as a flat spread curve up to the maximum maturity of the asset.
- The simplified assumptions for the calculation of expected loss are:
 - $EAD(i) = NA$, i.e. the exposure doesn't depend on market or amortization payments and is equal to the Notional amount. This assumption can be changed using the forward asset price to time point i for the $EAD(i)$
 - Recovery Rate within the $LGD(i)$ is constant for all periods.
 - Cumulative $PD(i)$ exists or can be represented by splitting to left and right RGP.
 - Period frequency is usually 3 months.
- Cumulative $PD(i)$ can be obtained from premiums of the spread curve by bootstrapping.

Bootstrapping: Calculation of PDs from CDS spread, using implications between premium and spread:

$$\text{Expected Loss (spread)} = \text{Present Value (premium)}$$

Example calculation with the following conditions:

Simple payment of 100 € at a time $t = 1$ year

Risk-free interest rate = 4%, CDS premium = 2%, Recovery rate = 45%

Calculation:

$$\text{Present value (premium)} = 100 * 0.02 / (1 + 0.04)^1 = 1.92 \text{ €}$$

$$\text{Expected Loss} = 100 * PD(1 \text{ year}) * (1 - \text{recovery rate})$$

$$PD(1 \text{ year}) = 1.92 / (100 * (1 - 0.45)) = 3.49\%$$

$$\text{Risk-free present value} = 100 / (1 + 0.04)^1 = \text{€ } 96.15$$

$$\text{Risk present value} = 96.15 - 1.92 = \text{€ } 94.23$$

Implied yield spread (spread on the evaluation of the curve)

$$\text{Risk present value} = 100 / (1 + 0.04 + \text{Yield Spread})^1 = \text{€ } 94.23$$

$$\text{Yield Spread} = 100 / 94.23 - (1 + 0.04) = 2.12\%$$

An extended description of the calculation of expected loss and the bootstrapping method is given in document 'Spread Calculator description v3.doc'.

5.3 Monte Carlo Simulation for obtaining Spread VaR

The representation of VaR distribution, with and without included spread risk, is given in Fig. 20. The simulation of spread VaR involves the following set of steps:

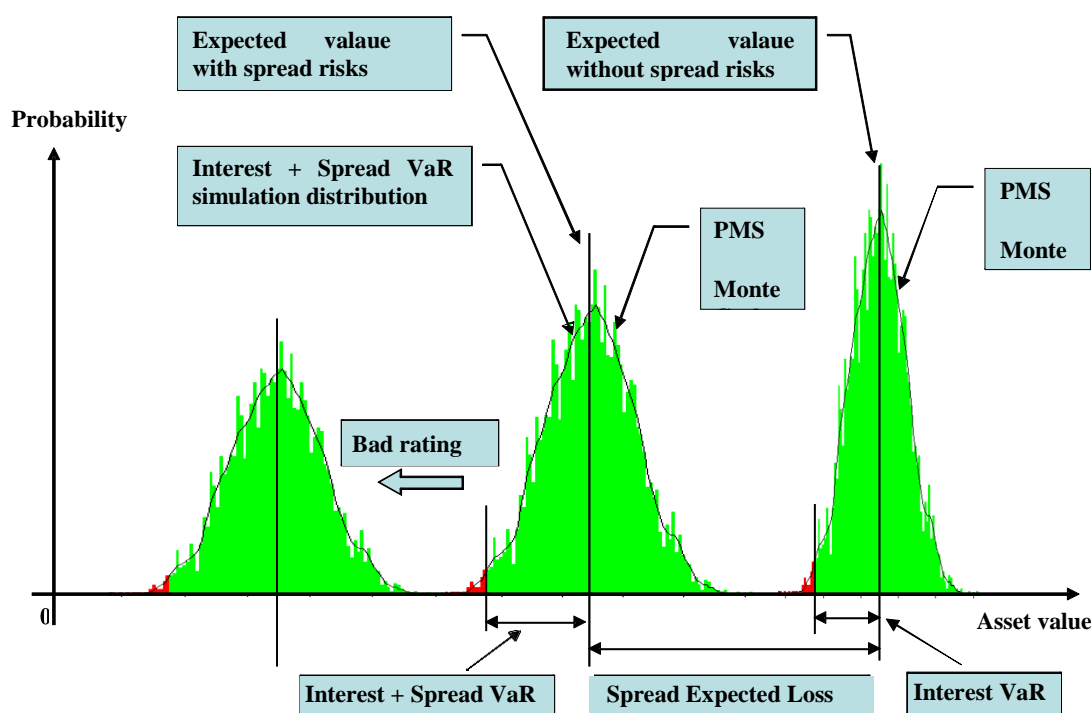


Fig.20 Representation of distributions with and without spread risk

- Inserting of the CDS curve factors as risk grid points into the risk grid points list.
- Calculation of market risk volatility and correlation matrix, bootstrapping the historical spreads of curve vertices into cumulative PDs per maturity.

- Construction of an expected loss simulation tree, based on formulas for the calculation of expected loss, described above or stated in the document 'Spread Calculator description v3.doc', before the simulation.
- Subtracting this tree from the simulation tree of the credit risk free asset, according to the following formula:

$$\text{Fair Value (asset)} = \text{Risk-free value (asset)} - \text{Expected Loss (asset)}$$

- Simulation of the asset, together with all other assets within the portfolio, and creating/e the Monte Carlo distribution for risky asset, including expected loss risk.
- Obtaining VaR results:

Interest rate VaR, spread VaR, total VaR, Incremental and Marginal VaR, etc.

- Spread VaR is calculated separately, by blocking all RGPs except RGPs (Cumulative PDs) from the bootstrapped spread curve.

The subtraction of the expected loss tree from the simulation tree of an asset involves two effects:

- The common distribution is moved to the left by the expected value of expected loss.
- The distribution becomes an additional volatility, because of the volatility of expected loss, i.e. the distribution includes correlated effects of interest rate and spread volatility.

The bad issuer rating causes higher CDS premiums and therefore higher cumulative PDs. Thus, the expected loss and the volatility of expected loss become higher, wherefore the distribution moves to the left, as shown in Fig. 20.

5.4 Spread VaR Simulation Example

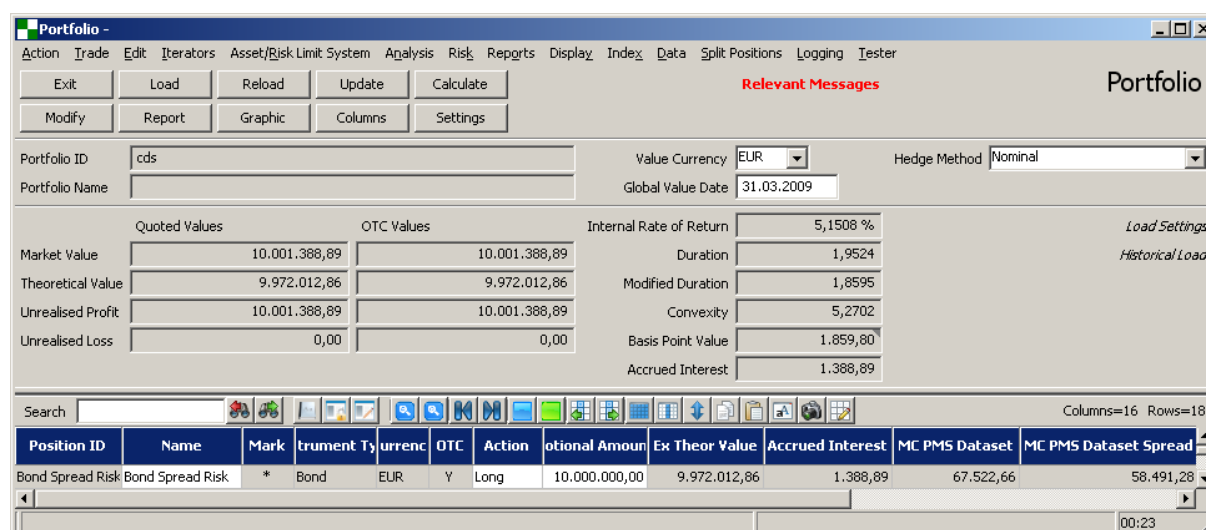


Fig. 21 Single fix bond in a portfolio

The calculation of spread VaR in the Monte Carlo simulation tree is illustrated using a simple asset example. The portfolio analysis from Fig. 21. includes only one fix bond position. The currency of the bond and the portfolio is EUR. The calculation is performed on the date 31.03.2009.

The bond exposes two payments, according to panel parameters:

- 500 000 EUR on 31.03.2010 as interest rate payment
- 10 500 000 EUR on 31.03.2011 final redemption of the notional amount of 10 000 000 EUR + 500 000 EUR interest rate payment

Bond: Bond Spread Risk / G-ID: Bond Spread Risk / P-ID: Bond Spread Risk

Action Calculate Rebuild Conventions Market Data Logging Regulatory Reporting Arrears

Exit Update Margins Show Yields **Bond**

Automatic Recalculation ☐ All Periods ☐

Name: Bond Spread Risk Notional Amount: 10.000.000,000 Coupon: 5,000000 %
 Serial Nr.: Bond Spread Risk Inflation Linked ☐ Face Value: 100,00 Agio: 0,000000 %
 Value Date: 31.03.2009 Currency: EUR Action: Long Convexity: 5,270171 Duration: 1,952384
 Start Date: 31.03.2009 Fix / Float: Fix Basis Point Value: 1.859,80 Modified Duration: 1,859546
 Effective Date: 31.03.2009 Yield Convention: ZeroBonds Consumer Price Index: Def. Inflation-Index Consumer Price Index Rate: 0,0000 %
 Issue Date: 31.03.2009 Maturity: 31.03.2011

Show Prices ☐ Show Stock ☒

Stock

	Clean Values	Dirty Values
Book Price	0,00	Paid -1.388,89
Interest on Book Price	0,000000 %	Market 10.001.388,89
Interest on Market Price	0,000000 %	Theoretical 9.972.012,86
	Market 10.000.000,00	Theoretical Profit/Loss 9.970.623,97
	Theoretical 9.970.623,97	

Accrued Interest: 1.388,89 Proceeds: 0,00 Profit/Loss: 10.000.000,00 Theoretical Profit/Loss: 9.970.623,97

Option Type: None Exercise Date: 30.09.2009 Volatility: 5,0000 % Option Premium: 0,00
 Cap / Floor: None Cap Rate: 0,0000 % Floor Rate: 0,0000 %

Market Segment ID: EUR Swap BEIR Curve: Select_a_BEIRCrV Yield Spread: 0,0000 bp Yield Spread Calculator

Start Date	End Date	Pay Date	Av. NAmount	Coupon	Amortisation	Rate Payment	Total CF	NPV	Proceeds	Discount Factor	Impl. Int. Rate	DT for DF
31.03.2009	31.03.2010	31.03.2010	10.000.000,00	5,000000 %	0,00	500.000,00	500.000,00	475.507,38	0,00	0,951015	5,1508 %	1,000000
31.03.2010	31.03.2011	31.03.2011	10.000.000,00	5,000000 %	10.000.000,00	500.000,00	10.500.000,00	9.496.505,47	0,00	0,904429	5,1508 %	2,000000

Fig. 22 Bonds data details

This cash flow structure will produce a very simple simulation tree, consisting of two nodes, for discount factor after 1 year and after 2 years, which coincide to RGP's of the EUR-Swap market curve, assuming it is credit risk free.

A spread risk calculation of the bond is included, because a CDS spread curve has been selected in field Credit Risk Curve in Fig. 23. Current premiums per maturity from the spread curve are given in browsers Spreads and PD. Cumulated and marginal PDs are obtained by bootstrapping the CDS premium. The ISDA Standard CDS bootstrapping method is used for the bootstrapping.

Using a periodic approach and 3 month periods, the expected loss is calculated to be 307 910,13 EUR,. The calculation of the expected loss is shown on Fig. 24. Periodic expected loss calculation, for the example, bond.

Spread Risk Calculator

Exit Accept Export Curve Calculate

Spread Risk Calculation

Name: Bond Spread Risk Currency: EUR On Index: ☐

Serial Number: Bond Spread Risk Position: Long Credit Risk Curve: BASF-AEUR-MMCM

Value Date: 31.03.2009 Notional Amount: 10,000,000,000 Base Date: 31.03.2009

Start Date: 31.03.2009 Market Segment ID: EUR Swap Underlying Type: CDS premium

Effective Date: 31.03.2009 Yield Spread: 0,0000 bp Recovery Rate: 40,00 %

Issue Date: 31.03.2009 Day-Count Convention: ACT/360

Maturity: 31.03.2011 Recovery Rate: 40,00 % Curve Frequency: Quart

Use Curve's Recovery Rate: ☒ Bootstrapping Method: ISDA Standard CDS

Used Recovery Rate: 40,00 %

	Theoretical Value	Theoretical Price	Spread / Rate
Excluding Spread Risk	9.972.012,86	99,72	5,000000 %
Expected Loss	307.910,13	3,08	1,659496 %
Including Spread Risk	9.664.102,73	96,64	3,340504 %

Implied Premium: 165,95 bp

Expected Loss Value: 307.910,13

Unwind Expected Loss: 307.228,11

Risk Curve BP Value: -1.803,19

Spreads & PDs ☐ Expected Loss ☒ Unwind Premium: 166,1000 bp Flatten Curve

Number	Offset	CDS Premium (bp)	Cumul. PD	Margin. PD
1	6m	167,887585	0,014034	0,014034
2	*1y	167,900000	0,027796	0,013762
3	*2y	166,100000	0,054228	0,026431
4	*3y	162,000000	0,078252	0,024024
5	*4y	153,127602	0,096955	0,018704
6	*5y	147,800000	0,115280	0,018324
7	*7y	137,200000	0,145408	0,030129
8	*10y	124,700000	0,180491	0,035083
9	*15y	140,800000	0,305546	0,125054

Fig. 23 Expected loss calculation: Bootstrapped CDS Curve

Spreads & PDs ☐ Expected Loss ☒ Unwind Premium: 166,1000 bp Flatten Curve

Number	Start	End	Cumul. PD	Margin. PD	Expected Loss	PV Expected Loss
1	31.03.2009	30.06.2009	0,007004	0,007004	42.021,26	41.512,85
2	30.06.2009	30.09.2009	0,014034	0,007031	42.183,87	41.157,66
3	30.09.2009	31.12.2009	0,021015	0,006981	41.885,20	40.348,73
4	31.12.2009	31.03.2010	0,027796	0,006781	40.687,68	38.694,59
5	31.03.2010	30.06.2010	0,034454	0,006658	39.948,38	37.517,44
6	30.06.2010	30.09.2010	0,041139	0,006685	40.109,27	37.198,52
7	30.09.2010	31.12.2010	0,047778	0,006639	39.831,57	36.480,03
8	31.12.2010	31.03.2011	0,054228	0,006450	38.698,81	35.000,33
					325.366,04	307.910,13
					EUR	EUR

Fig. 24 Periodic expected loss calculation for the example bond

The RGPs of the spread curve are included into the market risk correlation matrix, together with the RGPs of the evaluation curve and all other market risk factors. However, the bond example requires interest risk RGPs only for 2 years and only 2 RGPs from the spread curve. The Monte Carlo simulation module extracts only 6 RGPs from a large correlation matrix (given in table below). They are comprised of 4 interest rate RGPs and 2 spread curve RGPs.

Risk Grid Points within market risk matrix for the example bond

Global Monte Carlo matrix:							Risk Grid Points: 6	Volatility
	0	1	2	3	4	5		
0	1	0,5228	0,1195	0,3833	0,0497	0,1217	[EUR Y090 EUR Money]	0,0007%
1	0,5228	1	0,037	0,8427	0,0668	0,1676	[EUR Y180 EUR Money]	0,0022%
2	0,1195	0,037	1	0,0205	0,6456	0,0705	[EUR Z001 BASF-AEUR-MMCM]	12,5450%
3	0,3833	0,8427	0,0205	1	0,0349	0,1705	[EUR Z001 EUR Swap]	0,0083%
4	0,0497	0,0668	0,6456	0,0349	1	-0,0016	[EUR Z002 BASF-AEUR-MMCM]	8,1285%
5	0,1217	0,1676	0,0705	0,1705	-0,0016	1	[EUR Z002 EUR Swap]	0,1426%

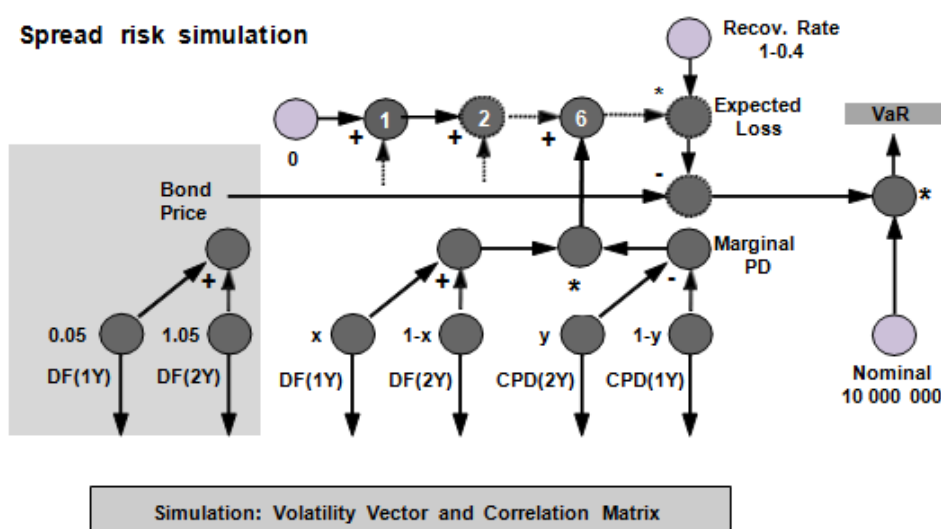


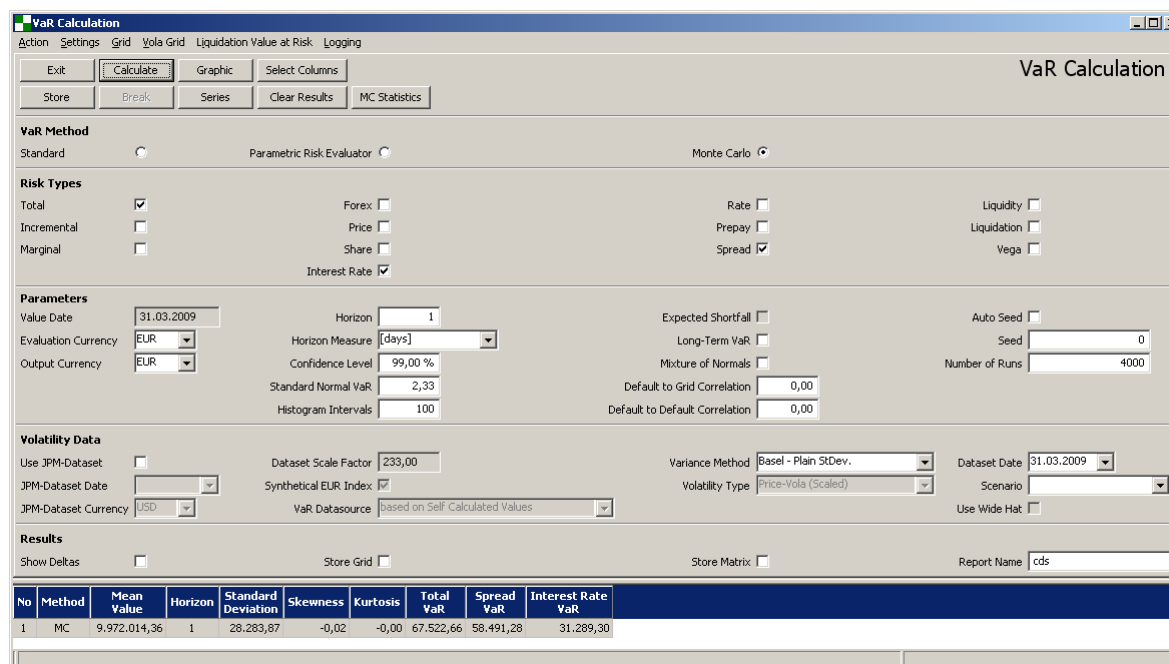
Fig. 25 Spread risk simulation tree: bond tree – expected loss tree

The simulation tree of the total risk (bond + expected loss) is shown in Fig. 25. The simulation tree follows the exact pricing formula of the bond, including the expected loss. The bond pricing tree is seen on the left and consists of two discount factor nodes, for 1 and 2 years. The expected loss part features the formula:

$$\text{sum}((\text{Cumulative PD}(i) - \text{Cumulative PD}(i-1)) * \text{Discount Factor}(i)) * \text{NA} * (1 - \text{Recovery Rate}))$$

There are a total of 8 summands in the sum, corresponding to 3 month periods within the bond's 2 years maturity. Every summand includes the term: Marginal PD(t) * Discount Factor(t). The marginal PD(t) is represented as the difference between Cumulative PDs at period begin and at period end, by splitting the left and the right spread RGP. The same splitting is done for the discount factor too. The sum is then subtracted from the bond tree. The result after the subtraction is multiplied by NA of 10 Mios and by 1 - RecoveryRate = 0,6. The representation of the simulation tree assumes fixed EAD = NA. The simulation structure should be changed if the development of the EAD along the time axis is recognized according to forward bond calculation.

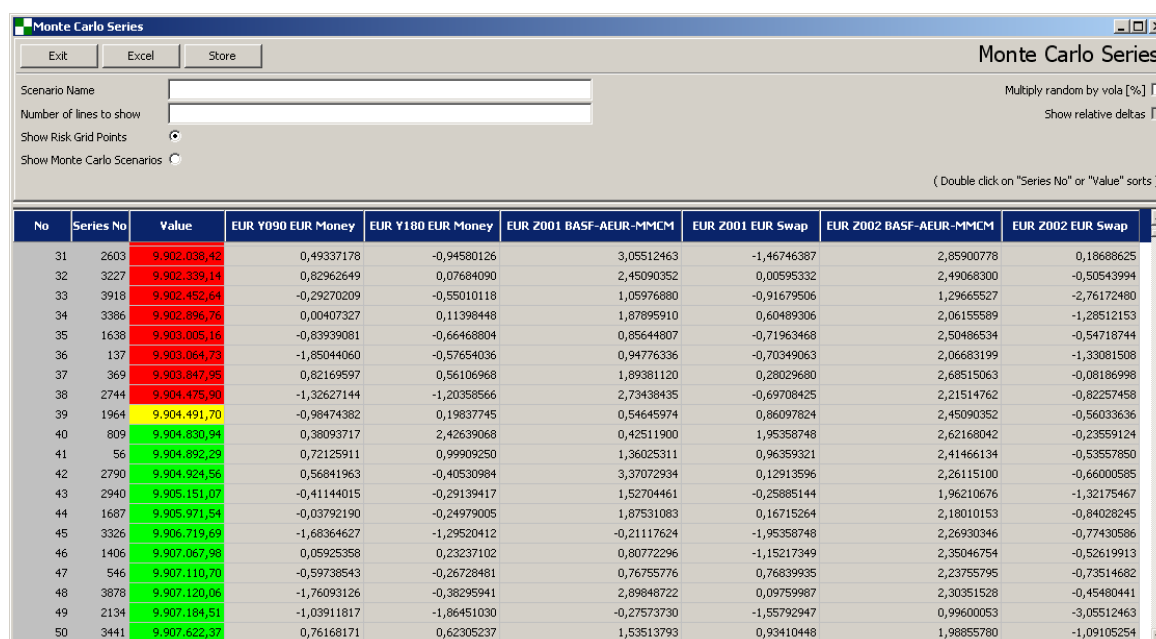
For every Monte Carlo scenario, the price distribution is obtained by the price calculation in the tree, starting on the VaR node and using a recursive procedure along the simulation tree.



No	Method	Mean Value	Horizon	Standard Deviation	Skewness	Kurtosis	Total VaR	Spread VaR	Interest Rate VaR
1	MC	9.972.014,36	1	28.283,87	-0,02	-0,00	67.522,66	58.491,28	31.289,30

Fig. 26 VaR results in PMS

Simulation results are presented in Fig. 26. A spread VaR of 58.491,28 is calculated in the tree, by blocking scenarios on interest rate nodes. In this case, only the CPD nodes are simulated; all other nodes remain static for all simulation runs.



No	Series No	Value	EUR Y090 EUR Money	EUR Y180 EUR Money	EUR 2001 BASF-AEUR-MMCM	EUR 2001 EUR Swap	EUR 2002 BASF-AEUR-MMCM	EUR 2002 EUR Swap
31	2603	9.902.038,42	0,49337178	-0,94580126	3,05512463	-1,46746387	2,85900778	0,18688625
32	3227	9.902.339,14	0,82962649	0,07684090	2,45090352	0,00595332	2,49068300	-0,50543994
33	3918	9.902.452,64	-0,29270209	-0,55010118	1,05976880	-0,91679506	1,29665527	-2,76172480
34	3386	9.902.896,76	0,00407327	0,11398448	1,87895910	0,60489306	2,06155589	-1,28512153
35	1638	9.903.005,16	-0,83939081	-0,66468804	0,85644807	-0,71963468	2,50486534	-0,54718744
36	137	9.903.064,73	-1,85044060	-0,57654036	0,94776336	-0,70349063	2,06683199	-1,33081508
37	369	9.903.847,95	0,82169597	0,56106968	1,89381120	0,28029680	2,68515063	-0,08186998
38	2744	9.904.475,90	-1,32627144	-1,20358566	2,73438435	-0,69708425	2,21514762	-0,82257458
39	1964	9.904.491,70	-0,98474382	0,19837745	0,54645974	0,86097824	2,45090352	-0,56033636
40	809	9.904.830,94	0,38093717	2,42639068	0,42511900	1,95358748	2,62168042	-0,23559124
41	56	9.904.892,29	0,72125911	0,99909250	1,36025311	0,96359321	2,41466134	-0,53557850
42	2790	9.904.924,56	0,56841963	-0,40530984	3,37072934	0,12913596	2,26115100	-0,66000585
43	2940	9.905.151,07	-0,41144015	-0,29139417	1,52704461	-0,25885144	1,96210676	-1,32175467
44	1687	9.905.971,54	-0,03792190	-0,24979005	1,87531083	0,16715264	2,18010153	-0,84028245
45	3326	9.906.719,69	-1,68364627	-1,29520412	-0,21117624	-1,95358748	2,26930364	-0,77430586
46	1406	9.907.067,98	0,05925358	0,23237102	0,80772296	-1,15217349	2,35046754	-0,52619913
47	546	9.907.110,70	-0,59738543	-0,26728481	0,76755776	0,76839935	2,25757955	-0,73514682
48	3878	9.907.120,06	-1,76093126	-0,38295941	2,89848722	0,09759987	2,30351528	-0,45480441
49	2134	9.907.184,51	-1,03911817	-1,86451030	-0,27573730	-1,55792947	0,99600053	-3,05512463
50	3441	9.907.622,37	0,76168171	0,62305237	1,53513793	0,93410448	1,98855780	-1,09105254

Fig. 27 Monte Carlo simulation series for the 5 RGPs

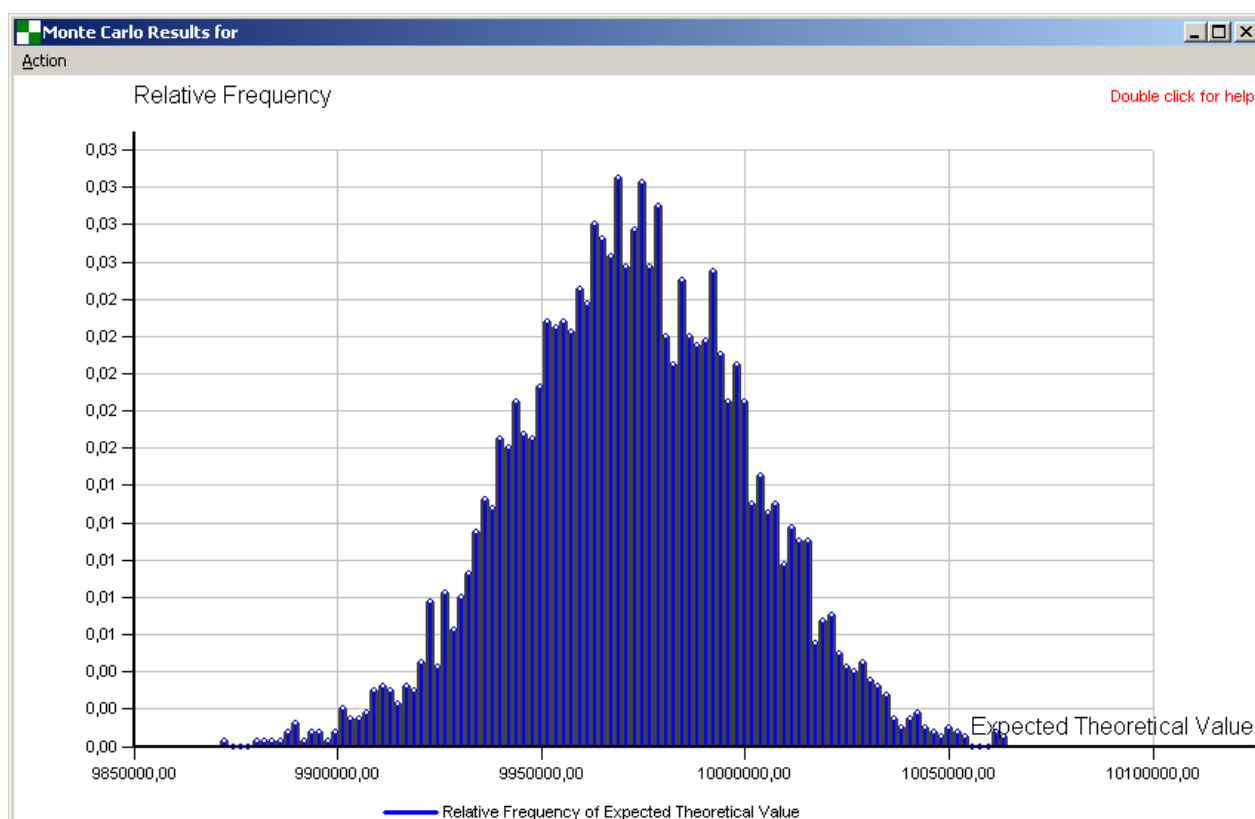


Fig. 28 Monte Carlo simulation distribution for total VaR

Fig. 27 and 28 show the simulation scenarios and the resulting distribution for total VaR.